

## True-Amplitude PS Prestack Time Migration via 5D Interpolation

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### Summary

Converted-wave prestack time migration is an important step in multi-component seismic data processing not only for its ability to produce a better PS image, but also as a preparation of the data for further analysis such as joint PP, PS inversion. Technically, its implementation is similar to, but not the same as, P-wave migration because the down-going P-wave and up-going S-wave travel at different speeds and are affected by anisotropy differently. Some technical issues, such as velocity and anisotropy parameterization, anti-aliasing, reflection amplitude preservation, random noise attenuation and 5D interpolation prior to converted wave (P-S) prestack time migration need to be handled appropriately in order to optimize the migrated image.

### Introduction

Imaging of converted-wave (PS) data requires some considerations that are different from P-wave data. Since the PS conversion point locations change substantially from top to bottom of a converted-wave seismic trace, the regular processing flow of NMO plus DMO and poststack migration can have difficulty achieving an optimal image. Kirchhoff prestack time migration is an ideal tool in converted wave imaging for areas with gentle lateral velocity variation. In areas where the velocity variations are more substantial, prestack depth migration would be more desirable (Kristiansen et al., 2003, Miao et al., 2005).

S-waves are typically more influenced by anisotropy in a medium than P-waves. Treatment of both horizontal transverse isotropy (HTI) and vertical transverse anisotropy (VTI) are important elements of the converted-wave processing flow. HTI produces shear-wave splitting of the P-S wavefield into separate P-S1 and P-S2 wavefields. Each of these wavefields are affected by VTI, which affects raybending, the lateral location of common-conversion points and residual moveout on the NMO-corrected gathers. The converted-wave prestack migration algorithm needs to account for the different intrinsic VTI anisotropy of P and S waves.

Like regular P-wave migration, migration aperture, amplitude weighting and migration operator aliasing are important factors for obtaining the correct migrated images. The maximum un-aliased frequency at a given slope of the migration operator can be derived using the same criteria as for P-wave migration (Lumley et al, 1994), but it is related to both P-wave and S-wave velocities. Preserving the relative amplitude of reflections in prestack seismic migration is not only important for obtaining the correct earth structure, but it is also important for subsequent AVO or AVAZ analysis. Based on the theory developed by Bleistein (2001), new formulas of true amplitude weights for both 2D common offset and 3D common-offset vector (Cary, 1999) based converted-wave prestack time migration (CPSTM) will be introduced in this paper. A good abstract presents technically correct ideas with a fresh and enlightening perspective.

### Theory and Method

Travel time computation is essential for converted-wave prestack time migration (CPSTM). In converted wave processing, the choice of parameterization is fundamental to streamlining the iterations of migration in order to update the velocities and anisotropy parameters. We follow the works of Li et al. (2001, 2007) and parameterize the double

square root equation of converted wave travel-time using four parameters: the converted wave RMS velocity  $v_c$ , the vertical velocity ratio  $\gamma$ , the effective velocity ratio  $\gamma_{eff}$  and the anisotropy parameter  $\chi$ .  $\chi$  is a non-physical parameter that is not directly related to the Thomson anisotropy parameters  $\varepsilon$  and  $\delta$  (Tsvankin and Thomsen, 1994). The double square root equation can be concisely written as,

$$t_c = \sqrt{t_{p0}^2 + \frac{\rho_s^2}{v_p^2} + \eta_p} + \sqrt{t_{s0}^2 + \frac{\rho_g^2}{v_s^2} + \eta_s} \quad (1)$$

where  $\rho_s$  and  $\rho_g$  are the distances from the shot and receiver to the image surface location respectively.  $v_p$  and  $v_s$  are P-wave and S-wave velocities, which are functions of  $v_c$  and the velocity ratios.  $\eta_p$  and  $\eta_s$  are anisotropy related P-wave and S-wave travel time terms. The advantage of using  $v_c$  and velocity ratios instead of P-wave velocity  $v_p$  and S-wave velocity  $v_s$  in CPSTM processing is that the travel-time of the PS-converted wave is not sensitive to variations in the velocity ratios. The effect of velocity ratio error on the move-out is much less than the effect of  $v_c$  error. In PS-converted wave data processing the values of the velocities obtained from stacking velocity analysis can be used in CPSTM. Thus, only the PS-converted-wave velocity needs to be estimated precisely.

In converted wave migration, we need to pay special attention to operator aliasing problems because of the large difference between down-going and up-going velocities. The correct frequency limit, which is dependent on  $v_p$  and  $v_s$ , can be derived based on the P-wave formula (Lumley et al, 1994, Abma, 1999, Wang 2004). The non-aliased high frequency limit is related to the derivative of travel-time ( $t$ ) to image surface location ( $\rho$ ),

$$f_{\max} = \frac{1}{2\Delta\rho} \left( \frac{\partial t}{\partial \rho} \right)^{-1}, \quad (2)$$

where  $\Delta\rho$  is the seismic trace spacing along the recording surface.

For the 3D case, from a simplified version of Equation (1) without anisotropy, the frequency limit becomes

$$f_{\max} = \frac{1}{2\Delta\rho} \left( \frac{\rho_s}{t_p v_p^2} + \frac{\rho_g}{t_s v_s^2} \right)^{-1}. \quad (3)$$

In Equation (3)  $\rho_s$  and  $\rho_g$  are the distance from the shot and the geophone to the image location respectively, and  $t_p$  and  $t_s$  are P-wave and S-wave travel-times.  $\Delta\rho$  is either inline interval or cross-line interval. For the 2-D case, the maximum un-aliased frequency is

$$f_{\max} = \frac{1}{2\Delta\rho} \left| \frac{x+h}{t_p v_p^2} + \frac{x-h}{t_s v_s^2} \right|^{-1}, \quad (4)$$

where  $x$  is the distance between image location and the trace mid-point and  $h$  is shot to receiver half-offset. Equation (4) is a simplified version of Equation (3). For P-wave data, there will be no differentiation of  $v_p$  and  $v_s$  and Equations (3) and (4) become P-wave anti-aliasing criteria.

Preserving the relative amplitude of reflections in prestack seismic migration is not only important for obtaining the correct earth structure, but it is also important for subsequent AVO or AVAZ analysis. Based on Bleistein's (2001) formula for the 3D case, the true amplitude weight is

$$W = \frac{H}{A|\nabla\phi|^2} \quad (5)$$

where  $H$  is the Beylkin's determinant,  $A$  is a geometrical spreading term, and  $\nabla\phi$  is the travel time derivative along ray-paths. In equation (5) and hereafter, the constants are neglected. For P-S wave propagation, assuming  $z$  is the image depth,  $\theta$  is the opening angle of down-going and up-going ray-paths,  $\gamma = v_p / v_s$  is the velocity ratio, and  $t_p$  and  $t_s$  are travel-times of P-wave and S-wave. The terms in equation (5) have following expressions

$$A = \gamma / (t_p t_s v_p^2),$$

$$|\nabla\phi|^2 = (1 + 2\gamma \cos\theta + \gamma^2) / v_p^2, \text{ and}$$

$$H = (1 + \cos\theta) \frac{z(\gamma^2 t_p + t_s)(\gamma^4 t_p^2 + t_s^2)}{\gamma_p^2 t_s^2 v_p}.$$

Therefore, we can derive 3D true amplitude weights as,

$$W = \frac{z(1 + \cos\theta)}{2v_p^3 \sqrt{1 + 2\gamma \cos\theta + \gamma^2}} \frac{(\gamma^2 t_p + t_s)(\gamma^4 t_p^2 + t_s^2)}{\gamma_p^2 t_s^2} \quad (6)$$

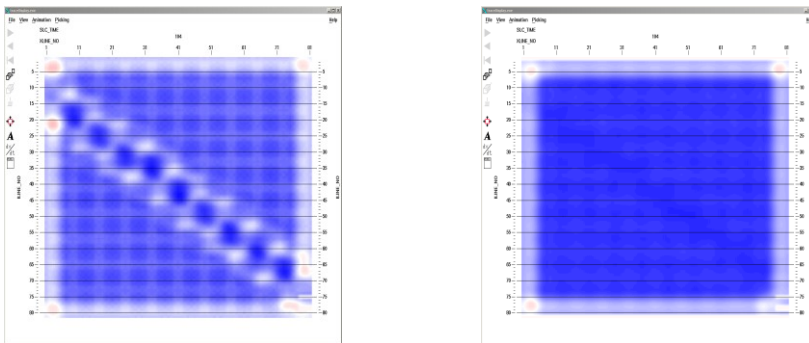
If  $\gamma$  is set to 1, which means single mode wave propagation, equation (6) and (8) simplify to P-wave true-amplitude migration weights, which are the same as that derived by Zhang et al, 2000.

Our CPSTM is based on common offset vector (COV) processing (Cary, 1999). A COV gather is a selection of traces with common inline-offset and common cross-line offset. Our 5D interpolation tool regularizes COV's in four spatial dimensions (ACP-X, ACP-Y, inline-offset and crossline-offset) to regularize observation geometry. The regularized and random noise attenuated COVs are extremely useful for obtaining true amplitude imaging of the subsurface since each COV forms a complete image of the subsurface for a particular offset and azimuth.

Velocity updating is critical for the imaging process which often requires several velocity picking and migration iterations. Converted-wave NMO velocities are typically used as initial migration velocities and then the velocities are updated in subsequent iterations along with effective  $V_p/V_s$ , which controls the effects of VTI on the lateral position of the conversion point as a function of offset.

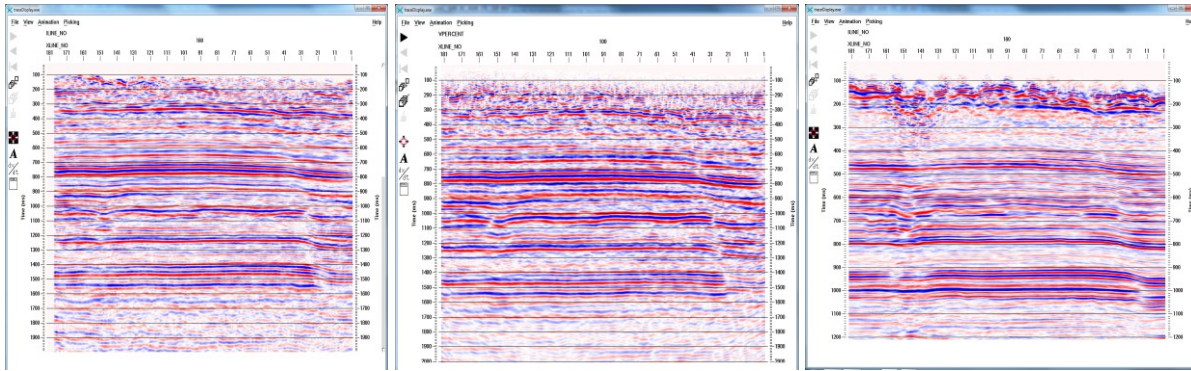
## Examples

Without 5D interpolation a footprint of the acquisition geometry can be imprinted on the data. This is especially easy to illustrate on a synthetic flat event. Figure 1 shows the importance of performing 5D interpolation into regularly sampled COV volumes before CPSTM since it successfully removes the imprint of the pattern of source and receiver lines on the time slice of a migrated flat event.



**Figure 1:** Time slice of migrated flat event (Left) without and (Right) with 5D interpolation.

Figure 2 shows a selected inline of PS and PP volumes from a land 3C/3D seismic survey with poststack and prestack migration of the PS data compared to prestack time migration of the PP data.



**Figure 2:** Selected inline from land 3C/3D survey: Left) PS data after poststack migration Middle) PS data after prestack migration, and Right) PP data after prestack migration.

A sharpening of the faults, an increase in coherence and some degree of noise attenuation from 5D interpolation and prestack migration brings the PS data closer in quality to the PP data.

## Conclusions

We have presented anti-aliasing formulas and true amplitude weights for converted wave prestack time migration that are fundamental for obtaining optimal prestack time migrated images. We reviewed issues in converted wave migration and presented a migration flow based on COV processing that optimizes P-S wave imaging.

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