Trace interpolation and elevation statics by conjugate gradients

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Summary

We present a conjugate-gradient based inversion to correct for surface statics and irregular trace spacing. The algorithm returns a solution to the extrapolated wavefield with complexity $O(n^{2.5})$. Convergence is fast in the wavelike region, and very slow in the evanescent region. Decimated traces are reconstructed even though no smoothing operator is applied, but recovered wavefields do not approach known source wavefields at low frequencies. We suggest that speed and accuracy of inversion by conjugate gradients can be improved through careful smoothing, or separate treatment of the wavelike and evanescent regions. Computing operators by series expansion for fast application during conjugate gradient iterations is suggested to optimize runtime.

Introduction

A wave equation inversion for acquired seismic data described in Ferguson (2006) recursively computes the extrapolated wavefield at depth using non-stationary phase shift operators (Ferguson and Margrave, 2002). The operator matrix is computed using an assumed velocity model and the wavefield at depth is derived using weighted damped least squares. This method is used to correct common shot gathers for topography and receiver statics, downward propagate the receiver wavefield through a heterogeneous near surface to a flat datum, and to correct for irregular spatial sampling in one inversion.

Full computation of the extrapolation matrix $P_{\Delta z}$ has complexity $O(n^2)$, where n is the number of spatial co-ordinates (Ferguson, 2006). Computing the least squares Hessian matrix requires multiplication of the extrapolation matrix by a weight matrix, followed by the adjoint extrapolation matrix, with complexity $O(n^3)$. Inversion of this matrix by Gaussian elimination also has complexity $O(n^3)$. In this paper we analyse the inversion of the Hessian by conjugate gradients. We assume that the phase shift operator perfectly models wave propagation, and observe rate of convergence and accuracy as a function of frequency.

Theory

A wave equation inversion for seismic data given by Ferguson (2006) simultaneously corrects for velocity variation in the near surface and irregular trace spacing using non-stationary phase shift operators. First we discuss here the development of these operators, and the application to statics and trace regularization. We will then discuss the conjugate gradient method as a means to speed the algorithm.

Non-stationary Phase Shift Operators

The phase-shift migration method of Gazdag (1978) models the propagation of a mono-chromatic wavefield through the subsurface as a function of a homogeneous velocity model. It gives a fast and exact solution to the scalar wave equation in homogeneous media (Gazdag and Sguazzero, 1984). To accommodate velocity variation in depth, the algorithm is run recursively on a sequence of constant velocity depth steps. That is, for each frequency ω, and each depth z, the extrapolated wavefield $\varphi_{z+\Delta z}$ is computed from φ_z by

$$\varphi_{z+\Delta z}(x) = \frac{1}{2\pi}\int \alpha_{\Delta z}(k_x, v_z)\int \varphi_z(x')\exp(ik_x(x'-x))dx'dk_x \ , \tag{1}$$

where $\alpha_{\Delta z}$ is a function of spatial wavenumber k_x and layer velocity v_z given by

$$\alpha_{\Delta z}(k_x, v_z) = \exp\left(\pm i\Delta z\sqrt{\left(\omega / v_z\right)^2 - k_x \cdot k_x}\right). \tag{2}$$

The choice of sign here varies with k_x. It determines which wavefield - upward or downward - is extrapolated, and attenuates spurious energy in the evanescent region, where the square root is complex, by ensuring the exponent is negative. Lateral velocity variation is modelled by computing a windowed reference wavefield with respect to each velocity in our model, and superimposing the results (Margrave and Ferguson 1999). We will denote by $P_{\Delta z}$ any of these one-way operators that shift a wavefield downward by Δz.

Statics and Trace Regularization

Ferguson (2006) presents an application of these phase-shift operators to correct for surface statics and irregular trace spacing. Acquired seismic data is modelled recursively as follows: given a recorded wavefield vector φ_z at depth z, we assume that $\varphi_z = W_e P_{-\Delta z}\varphi_{z+\Delta z} + \zeta$, where $P_{-\Delta z}$ is an upward phase shift, as in Equation 1, W_e is a weighting operator that models irregular trace spacing and topography (Reshef, 1991), and ζ is an additive noise term. The least-squares approximation of $\varphi_{z+\Delta z}$ is recovered by minimizing the misfit function

$$M(\varphi) = \left\| W_e\left(P_{-\Delta z}\varphi - \varphi_z\right)\right\|^2 + \varepsilon\left\| W_m\left(\varphi - \varphi_m\right)\right\|^2 . \tag{8}$$

Here W_m is a smoothing operator, φ_m is the a priori information on the model parameters (Tarantola, 2005), and ε is a user parameter that controls the amount of smoothing (Menke, 1989). *M* is minimized when the normal equations are satisfied, so we recover $\varphi_{z+\Delta z}$ by solving the linear system

$$\left[S_{-\Delta z}\right]\varphi_{z+\Delta z} = \left[P^*_{-\Delta z}W_e P_{-\Delta z} + \varepsilon W_m\right]\varphi_{z+\Delta z} = P^*_{-\Delta z}W_e\varphi_z + \varepsilon W_m\varphi_m . \tag{9}$$

where $P_{-\Delta z}^*$ is the adjoint of $P_{-\Delta z}$. If we consider these operators to be matrices, then the computing cost of recovering $\varphi_{z+\Delta z}$ is dominated by the cost of computing then inverting the matrix $S_{-\Delta z}$. Ferguson (2006) derives a series approximation of $S_{-\Delta z}$ to speed up computation of the matrix, and we consider using conjugate gradients to speed inversion.

Conjugate Gradients

The conjugate gradient method is an iterative algorithm used to approximate a solution x to a linear system Ax = b. In our case it can be used to recover the source wavefield $\varphi_{z+\Delta z}$ from Equation 9. Inverting an nxn matrix by Gaussian elimination has complexity $O(n^3)$ (Strassen, 1969), whereas solving the system by conjugate gradients can return an acceptable approximation in about √n iterations of complexity $O(n^2)$ each, provided the matrix is well-conditioned (Burden and Faires, 2001). A matrix is well-conditioned if it is not sensitive to rounding errors.

Method and Example

An arbitrary source wavefield of n=256 traces with n temporal samples (Figure 1a) is Fourier transformed in time, and band limited, from 4 to 125 Hz. The synthetic data φ_0 (Figure 2a) is computed from the resultant monochromatic wavefields according to $\varphi_0 = W_e P_{-100}\varphi_{100} + \zeta$, where P_{-100} is NSPS (Margrave and Ferguson, 1999) computed with respect to the velocity model in Figure 1a, W_e is the identity matrix with a random selection of approximately 30% of the diagonal entries set to 0, and φ_{100} is the source wavefield shown in Figure 1a. Here Δz=100m to exaggerate the visual impact of the phase shifted data (Figure 2a), since we will be restricting our attention to a single depth step. The effects of smoothing are not considered here, so ε in Equation 9 is set to 0. We then apply Matlab's Preconditioned Conjugate Gradient (pcg) algorithm to Equation 9 with no preconditioning, and using the zero vector as an initial guess. We run pcg for $\sqrt{n}$ iterations, or until the relative residual error falls below a tolerance of 10^{-6}.

The algorithm converges to within the tolerance fastest in the high frequencies (~6 iterations) where our data does not cross into the evanescent region. Convergence generally slows as frequency decreases because our operator is poorly conditioned at low frequencies. This poor conditioning results from small operator eigenvalues in the evanescent region, where the phase shift is a real exponential with negative exponent. The algorithm did not converge to within the desired tolerance at frequencies below 90Hz, although the relative residual error was acceptably small.

The resulting recovered wavefield is shown in Figure 2b. The source wavefield is roughly recovered, with the image properly positioned, and many missing traces recovered. There is some artifact around the bright areas of the image, and trace interpolation is less effective where there are large gaps in trace coverage. Our recovered wavefield does not agree strongly with the known source wavefield since no smoother was applied. Clearly some smoothing is required to recover the ideal solution. However, we expected that no smoothing would result in poor regularization, and this was not the case. Some regularization results from the search directions prescribed by the conjugate gradient method, although additional iterations don't increase the desirability of the solution.

Conclusions

Using NSPS as our model of wavefield propagation, we find that the conjugate gradient algorithm applied to the least squares minimization problem gives a rough solution to the extrapolated wavefield in $\sqrt{n}$ iterations and no significant improvement is gained from subsequent iterations. Convergence is fast in the wavelike region, and slow in the evanescent region, and we postulate that the slow convergence is caused by small operator eigenvalues from the evanescent part of the wave extrapolator $\alpha_{\Delta z}$, which cause the Hessian to be almost singular. Solution damping could be achieved through the use of a nontrivial smoothing operator W_m, as in Smith et al. (2009) and Ferguson (2006), or we might attempt to treat the wavelike and evanescent regions separately.

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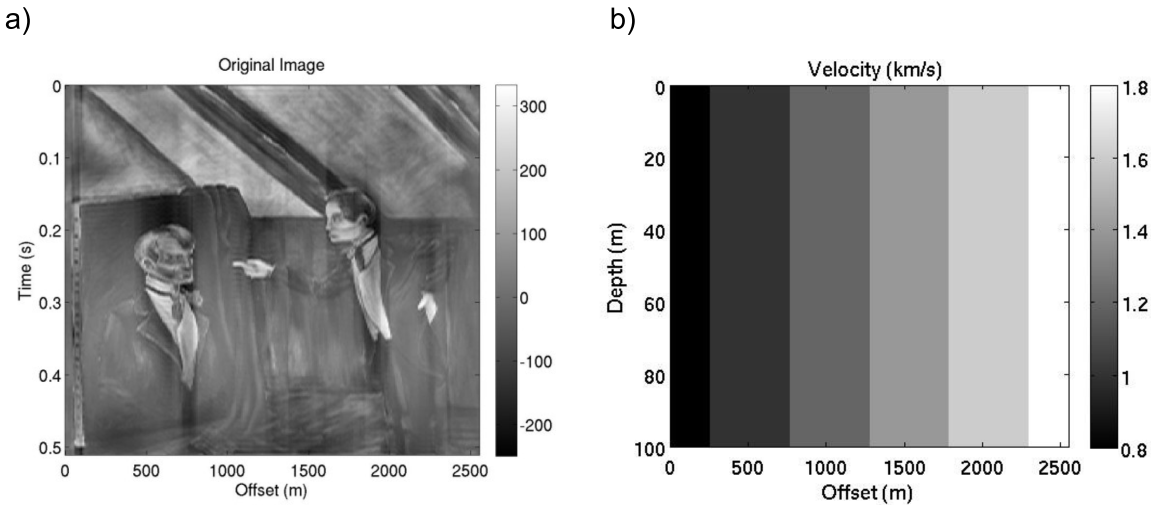

Figure 1: a) Band limited source wavefield, b) Laterally varying velocity model

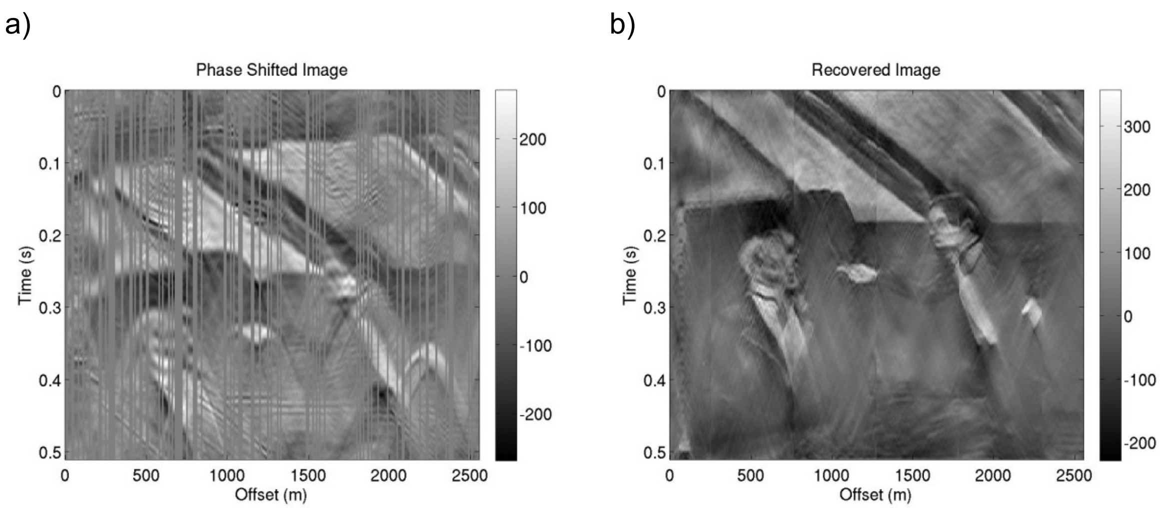

Figure 2: a) Phase shifted image with 30% trace decimation. b) Recovered image