Wavelets Transforms: Time – Frequency Presentation

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Summary

Upstream industry has to make fashionable the potential of wavelet analysis which plays a pivotal role in exploration and production of hydrocarbon to enhance R/P ratio of nations with a view to fuel security of the world. Non-Stationary statistical Geophysical Seismic Signal Processing (GSSP) is of paramount importance for imaging underground geological structures and is being used all over the world to search for petroleum deposits and to probe the deeper portions of the earth. Wavelet analysis , known as a mathematical microscope, has scope to cope with non stationary signal to delve deep into geophysical seismic signal processing and interpretation for oil & gas exploration & production , Petrophysical imaging for oil & gas reservoir ,Advanced Seismic Stratigraphy: A Sequence-***Wavelet*** Analysis Exploration-Exploitation,high resolution subsurface imaging and modeling for complex earth media. Extraction of informations from signals follows time-frequency atoms representation & transformations for rectification of uncertainty principle limitation by wavelet first,second & third generation.Diplet-based seismic processing method include decomposing one or more original multi-dimensional volumes into a collection of diplets, wherein each diplet comprises information about spatial location, orientation, amplitude, wavelet, acquisition configuration, coherency and anisotropic velocity model in migration.Amplitude Versus Frequency(AVF) - The resolution limit of seismic data is a complex issue that involves not only wavelet frequency ,phase characters,and data quality (S/N ratio) ,but criteria on how to measure resolvability.Wavelet based AVO(Amplitude Variation with Offset) is applied for precise subsurface imaging in anisotropic processing . **Third generation wavelet -** a Complex Finite Ridgelet Transform (CFRIT),to achieve the forensic dissection, morphological features from micro/nano scalar of surface topographic data . Current status of **Wavelet Transforms are** Diplet, Ridgelet(Radon & Wavelet), slantlet, Curvelet, phaselet,beamlet, contourlet,caplet, Seislet .

Theory/Methods: The *minimum phase* wavelet has short time duration and a concentration of energy at the start of the wavelet. It is zero before time zero (causal). An ideal seismic source would be a spike (maximum amplitude at every frequency), but the best practical one would be minimum phase. It is quite common to convert a given wavelet source into its minimum phase equivalent, since at several processing stages (e.g. predictive deconvolution) work best by assuming that the input data is of minimum phase. The *maximum phase* wavelet is the time reverse of the minimum phase and at every point the phase is greater for the maximum than the minimum. All other causal wavelets are strictly speaking *mixed-phase* and will be of longer time duration. The convolution of two minimum phase wavelets is minimum phase. The *zero-phase* wavelet is of shorter duration than the minimum phase equivalent. The wavelet is symmetrical with a maximum at time zero (non-causal). The fact that energy arrives before time zero is not physically realizable, but the wavelet is useful for increased resolving power and ease of picking reflection events (peak or trough). The convolution of a zero-phase and minimum phase wavelet is mixed phase (because the phase spectrum of the original minimum phase wavelet is not the unique minimum phase spectrum for the new modified wavelet) and should be avoided. A special type of wavelet often used for modelling purposes is the *Ricker* wavelet which is defined by its dominant frequency. The *Ricker* wavelet is by definition zero-phase, but a minimum phase equivalent can be constructed. The *Ricker* wavelet is used because it is simple to understand and often seems to represent a typical earth response.

Most of the signals in practice are time-domain signals in their raw format. When we plot time-domain signals, we obtain a *time-amplitude representation* of the signal. This representation is not always the best representation of the signal for most signal processing related applications. In many cases, the most distinguished information is hidden in the frequency content of the signal. The *frequency spectrum* of a signal provides the constituent frequencies present in the signal. When the *time localization* of the spectral components is needed, a transform giving the *Time-Frequency representation* of the signal is needed. The Wavelet transform is a transform of this type. It provides the time-frequency representation.

To analyze non-stationary signals, the technique of wavelet transform is needed, i.e. whose frequency response varies in time. Although the time and frequency resolution problems are results of a natural physical phenomenon and exist regardless of the transforms used, it is possible to analyze any signal by using an alternative approach called the *Multi-resolution Analysis (MRA)*. It analyzes the signal at different frequencies with different resolutions. Every spectral component is not resolved equally as is the case in the STFT (Short Time Fourier Transform). It is to be noted that *MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.* The Continuous Wavelet Transform (CWT) can be expressed as:

$$CWT_x^{\Psi}(\tau,s) = \boldsymbol{\Psi}_x^{\psi}(\tau,s) = \frac{1}{\sqrt{|s|}}\int x(t)\cdot\psi^*\left(\frac{t-\tau}{s}\right)dt$$

The computation of the above equation for CWT can be performed systematically in the following steps: Step 1: The wavelet is placed at the beginning of the signal, and set s=1 (the most compressed wavelet);Step 2: The wavelet function at scale ‘1’ is multiplied by the signal, and integrated over all times; then multiplied $1/\sqrt{s}$; Step 3: Shift the wavelet to $t=\tau$, and get the transform value at $t=\tau$ for $s=1$; Step 4: Repeat the procedure until the wavelet reaches the end of the signal; Step 5: Scale s is increased by a sufficiently small value, the above procedure is repeated for all *s*; Step 6: Each computation for a given *s* fills the single row of the time-scale plane; Step 7: CWT is obtained if all s are calculated.

In seismic data processing and analysis, scale is very important to extract information from signal. In terms of frequency, low frequencies (i.e., high scales) correspond to a global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal (that usually lasts a relatively short time). In terms of mathematical functions, if f(t) is a given function f(st) corresponds to a contracted (compressed) version of f(t) if $s > 1$ and to an expanded (dilated) version of f(t) if $s < 1$. However, in the definition of the wavelet transform, the scaling term is used in the denominator, and therefore, the opposite of the above statements holds, i.e., scales $s > 1$ dilates the signals whereas scales $s < 1$, compresses the signal. The highly efficient softwares for all aspect of reservoir characterization and simulation using the latest scientific tools (wavelets, fractals, etc.), the continuous wavelet transform, not only provides a proper multi-scale representation, hosting the complexity, but it also facilitates the introduction of a multi-scale analysis and subsequent (mathematical) characterization, unraveling the scaling structure of the well-log's complexity.

Tuning Effect : A phenomenon of constructive or destructive interference of waves from closely spaced events or reflections. At a spacing of less than one-quarter of the wavelength, reflections undergo constructive interference and produce a single event of high amplitude. At spacing greater than that, the event begins to be resolvable as two separate events. The tuning thickness is the bed thickness at which two events become indistinguishable in time, and knowing this thickness is important to seismic interpreters who wish to study thin reservoirs. The tuning thickness can be expressed by the following formula:
$Z = V_I/2.8\ f_{max}$, , where Z = tuning thickness of a bed, equal to 1/4 of the wavelength, V_I = interval velocity of the target , f_{max} = maximum frequency in the seismic section.
The equation assumes that the interfering wavelets are identical in frequency content and are zero-phase and is useful when planning a survey to determine the maximum frequency needed to resolve a given thickness. Spatial and temporal sampling requirements can then be established for the survey. ***Singularity detection** of the thin bed seismic signals with wavelet transform:* The location of singularities may be detected by local maxima of the wavelet transform modulus. The digital modeling and focusing process to wavelet transform of the reflecting seismic signals have been done. It has been found that the locations of singularities, after wavelet transform is performed, are only affected by two factors (their original locations and the seismic wavelet length) irrespective of the shape the wavelet. The wavelet length can be determined according to the wavelet transform results to detect thin bed with resolution of $\lambda / 32$. The singularities have been recovered with improved resolution of the seismic section by real data processing. Thus, the wavelet transform provides better resolution of the thin beds which, in turn, gives better picture of the seismic stratigraphy.

Results : Wavelet transforms yield high resolution subsurface and petrophysical imaging of hydrocarbon reservoirs. **Petrophysical Imaging:** Geophysical Well-log data are non-stationary in nature and show cyclic trends and abrupt changes of rock properties. Wavelet analysis is more effective for analyzing non-stationary signals in which, a signal can be represented as the sum of different frequency components with different resolutions. Wavelet transform is called the microscope of mathematics; it can divide signals into many scale elements which can complect together. Wavelet analysis is a multi-resolution framework and, thus, it is well suited for up scaling rock and flow properties in a multi-scale heterogeneous reservoir. The compact support property of the wavelet transform assures efficient computation. Choice of regularity provides a flexible way to control the smoothness of the resulting up scaling properties. Because the up scaled property images obtained from wavelet transform capture the characters of the original property fields, the predicted performance from up scaling property fields' matches well with that from the original fine-scale property fields. Geophysical well-log (bore-hole) data represent the rock physical properties as a function of depth measured in a well. They aid in demarcating the subsurface horizons, identifying abrupt changes in physical properties of rocks and locating cyclicity in stratal succession. Since wavelet transformations can better identify the abrupt changes in cyclicity common in nature, they become important tools for seismic stratigraphy. Currently spectral decomposition methods (Continuous Wavelet

Transform, Matching Pursuit decomposition, Discrete Wavelet Transform) are used to detect hydrocarbon zones. The wavelet transform is a multi-scale operator and is well known to point out singularities in the analyzed signal. The way in which the wavelet transform analyses the signal can be compared to the geoscientist's interpretative behaviour, which requires to look at the signal at different scales (frequency range) to detect breaks (i.e., major events) and heterogeneity for characterizing the trends. In order to measure multi-fractility of any signal, the Holder Exponents and Singularity Spectrum attributes are computed; whereas the local wavelet attributes (amplitude, phase and scale) are computed for geological characterization. The new development of fast wavelet transform (WT) is considered to be a revolutionary breakthrough in signal analysis; therefore, the wavelet transform will be applied to the one-way wave propagation methods to develop efficient 3D imaging and modeling methods. The cross-breeding of these two new developments has the potential of revolutionizing modeling and imaging techniques for complex earth media.

Conclusion : Seismic Spectral Blueing (SSB) : The wavelet analysis of seismic data with new paradigm of non-stationary signal processing will enable to extract more and more information from analytic signal. **MATLAB Wavelet Toolbox and** *Signal Processing* **Toolbox** is to be employed for wavelet analysis in this research work. **Mathematica- *Wavelet Explorer*** will also be employed for different types of wavelet analysis. Further, mathematical transformations are needed to apply on signals to obtain frequency, amplitude and phase compositions. The transforms which are needed to be applied are: Hilbert Transform, Radon Transform, TT-Transform and S-Transform; and Wavelet Transform (Continuous Wavelet Transform, Discrete Wavelet Transform and Complex Wavelet Transform). In wavelet transform the specialized features are to be analyzed for Curvelet, Ridgelet and Diplet. The Complex Wavelet Transform is being employed to minimize the limitations involved such as, shift sensitivity, poor directionality and absence of phase information. These extensions are highly redundant and computationally intensive. Observed behaviour of reflectivity data obtained from wells shows that, in a global sense, the higher the frequencies the higher the amplitude. We refer to this as the spectrum being blued. During the processing of seismic data the amplitudes are often whitened. Spectral shaping of seismic data using SSB can enhance resolution without boosting noise to an unacceptable level. SSB sections are usually easier to interpret and reveal more information about the subsurface. Small scale faulting, not seen on the original section, is often observed. Such additional information can be useful during well planning.

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