Event-directed Spectrum Extrapolation for Upsampling Seismic Traces

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Summary

In this paper, a practical and stable seismic trace interpolation is presented. The method is based on realizing that the spatial spectrum component is closely related to the dip parameters used in Radon transform and that the dip parameter is independent from the frequency within a given data set. Therefore, in frequency domain, a portion of the low un-aliased frequency band can be used for the dip parameters' estimation. Because the estimator uses multi frequencies the dip parameter estimate should be robust. By simply multiplying the frequency and dip parameters one can obtain the spectrum component with a designed Nyquist interval, which in turn, can be used for building a weighting function for spectrum extrapolation for the upsampling problem.

Introduction

With requirements of higher and higher resolution in seismic imaging, seismic trace interpolation has become an important procedure of seismic data processing. Trace interpolation for data that contains relatively flat events is quite simple and temporal-spatial domain interpolation algorithms, such as linear, cubic and polynomial fitting, can fulfill this task (e.g. Wang, X., 2002); otherwise sophisticated methods need to be employed in order to perform interpolation along event direction, e.g. cross-correlation (Malloy, J. 1989), Radon transform (Nurul and Verschuur, 1995), or a dip-filter method (e.g. Ji and Claerbout, 1997). Since last decade, a methods based on spectrum extrapolation technique have gained very interest in signal and image reconstruction (e.g. Ferreira, 1996). The key point of the spectrum extrapolation method is that: assuming the spectrum of design traces is approximately available, and this spectrum is used as *a priori* information to guild interpolation that makes energy concentrated along events distribution. How successful this spectrum extrapolation based interpolating method is dependent on how well the knowledge of the *a priori* spectrum information can be obtained. Originally the spectrum extrapolation method was developed for band limited missing data reconstructing by constraining the spectrum within known narrow band (Papolis, 1975). Youla extended Papolis's method to multi-band limited data, i.e. POCS method (Youla, 1978). By recognizing that partially missing signal does not significantly change the pattern of the spectrum, Cabrera (1991) developed a minimum weighted norm extrapolation method by using the power spectrum of the known signal as the main criteria for the weighting function to guild the energy concentrated proportionally to spectrum distribution. Cabrera's method can be considered as an extension of POCS method. When both methods are implemented with iterative conjugate gradient technique, both methods are formulated exactly same except for the difference of the weighting function (Strohmer, 1997) and if setting zeros for those elements that correspond to ignorable spectrum energy, both algorithm convergences to exactly same one. However, it is important to be aware that if the multi-band intervals are known, the POCS method works more robust than Cabera's minimum weighted norm method because the accuracy and stability of these methods depends on the ratio of band width used for the weight and Nyquest interval (Tarczynki, A., 1997). Seismic application of this spectrum extrapolation method can be found in works by Abma (2003) for FOCS method and Liu and Sachhi (2004) for Cabrera's minimum weighted norm method. The advantage of the spectrum method over other standard methods, such as FX predict filter (Spitz, 1991), is its more flexible for application, e.g. irregular sampling and higher dimension applications. While this method is now intensely applied in signal processing field for missing signal reconstruction, its application for upsampling is still limited. When applying for seismic trace upsampling by treating interpolated traces as missing data, the effective spatial Nyquist interval does not coincide with the designed Nyquist interval, and as the result, the spectrum from available data cannot be used as constraining/weighting criteria because of its strong aliasing. Therefore, the problem of upsampling now is transferred to how to find a spectrum that has same pattern as that of designed spectrum. Zwartjes and Sachhi (2007) based on an idea that the un-aliased the low frequency portion spectrum of the original data has a similar pattern to that of designed data. Therefore, weighting function can be build by extrapolating this low frequency portion spectrum to whole frequency range. However, when designed spatial spectrum is still aliased, such an extrapolation meets some difficulty. Naghizadeh and Sacchi (2009) gave a method based on a multi-step interval prediction filter for spectrum estimation, but this method also has a problem when the designed spatial spectrum is aliased. Moreover, a large step interval prediction filter needs a long data sample length, which does not favor to windowed trace interpolation that is required for linear events assumption.

In this paper, a practical and stable spectrum estimator is presented. The method is based on the realization that the spatial spectral component is closely related to dip parameter used in Radon transform, and that the dip parameter is independent from frequency within given data set. In the frequency domain, the low, un-aliased frequency band can be used for dip parameter estimation. Because the estimator uses multi frequency, the dip parameters estimate should be robust. By simply multiplying the frequency and dip parameters one can obtain the spectrum component with a designed Nyquist interval, which in turn, can be used for building a weighting function for spectrum extrapolation for the upsampling problem.

Method

Although natural spectrum related to seismic spatial events are not exactly band-limited, however, the band-limited characters can be obtained by ignore those frequency components that correspond to a negligible energy content and the effect can be visually imperceptible. By the filtering process in this way leads to a catalog of spectrum extrapolation techniques for seismic data reconstruction. An example of application of this technique is a recent work by Naghizadeh and Sacchi (2009). If events related spatial spectrum component $k_j, i = 1,2,\cdots,q$ (q is much less than the number of whole spectrum components) are found, then Fourier reconstruction can be formulated by first using

$$d(x_h, f) = \sum_{j=1}^{q} D(k_j, f) e^{-ik_j x_h} \qquad (1)$$

to define the spectral components $D(k_j, f)$,

$$d(x,f)=\sum_{j=1}^{q}D(k_j,f)e^{ik_jx} \qquad (2)$$

then to reconstruct data at $d(x,f)$. Another example is MWNI (Liu and Sachhi, 2004), which effectively apply $D(k_jf)$ as a weight function. Such a band-limited extrapolation works actually as constraints the reconstructed energy that concentrates within a limited spatial frequency bands. Comparing equation (1) with Radon Transform (Kostov, 1990),

$$d(x_h,f)=\sum_{j=1}^{q}D(p_j,\omega)e^{-i\omega p_jx} \qquad (3)$$

they are identical if $\omega p=k$. Parameter *p* in Radon Transform has a defined physical meaning; it is the dip of the event and therefore, the directions constrained by spectrum are event-directions. Therefore, estimate correctly each band of spectrum is of most important to spectrum extrapolation. However, when the event is spatially aliased, finding either *k* or *p* becomes difficult, especially for upsampling problem where spectrum is strongly aliased.

The key of our event-directed spectrum estimator is based on the fact that spatial spectrum component *k* at each ω can be calculated from given *p* that is independent from ω. The spectrum estimate can be firstly based on the original data of selecting a temporal frequency band, within the band all seismic events that is not spatial aliased, and then to estimate dip parameter *p*. The estimated *p* is scaled by ω to obtain the *k* components based on a designed spatial Nyquist frequency interval and finally, applying *k* as a spectrum extrapolation weight function makes the method better suited for upsampling interpolation.

Applications

As discussed in introduction all spectrum based methods depend on how well the designed spectrum estimation and therefore, applying the event-directed spectrum estimate method can make all of those methods more efficient and robust.

POCS method application

The POCS method is based on knowledge of a multi-band interval estimate, which can be defined by setting a threshold for power spectrum (e.g. Abma, 2003). When the spectrum is aliased, such a threshold setting approach does not work. With an event-directed spectrum estimator, the multi-band intervals can easily defined, which makes the method work for upsampling interpolation.

MWNI method application

The crucial point of MWNI method is to use the available data power spectrum as a weight function to tune reconstructed energy distributed along an event direction distribution. However, the optimal weight function is the one that is obtained from the designed spectrum and therefore iteratively update of the weight function is required. Because the weight function is used only for constringing energy’s distribution, the most important factor is the shape and relative values of weight function rather than the values of power spectrum itself. Therefore, smooth of power spectrum is applied for better shape. With an event-directed spectrum estimator, the optimal weight function can be easily created and no need to update it, which makes the interpolation more efficient.

Fourier reconstruction method application

Applying event-directed estimated *k* components of spatial spectrum to formulae (1) and (2) can result in the same procedure as that in Naghizadeh and Sacchi's works (Naghizadeh and Sacchi, 2009). In their work, the spatial spectrum is estimated at each frequency by a multi-step AR model. By using low frequency with large sample interval to predict the spectrum component at high frequency, the requirement of data length can be in excess of practical data length, and the aliasing problem can still exist. The advantage of our spectrum estimator is that firstly the estimation does not need to be carried out at each frequency, and second, no large step interval is needed.

Discussions

As mentioned earlier, seismic trace interpolation needs to follow seismic events. Interpolation in spatial frequency domain has aliasing problem, which is not shown in a spatial domain AR model method such as FX predict filter method (Spitz, 1991, Porsani, 1999). However, in order to predict a fractional step ahead, the prediction filter has to be calculated at a corresponding low frequency. Usually, spatial distribution of energy can vary with frequency, especially for band limited signal, which can lead to the filter coefficients not optimal and the amplitude of interpolated traces can have a scaling problem. Actually, if the convolution of FX interpolation method is transferred to spatial domain, this method has a similar physical meaning to the spectrum extrapolation method. Therefore, when the AR model is used for spectrum estimate (e.g. Naghizadeh and Sacchi, 2009), interpolating can follow event directions in the same way as that of FX's. By estimate dips instead of directly spatial spectrum is more convenient for high dimension application (e.g. Trad, *et al*, 2008), because dimension for dip is a function of spatial variable.

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