Geostatistics with Locally Varying Anisotropy

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Introduction

Since the origin of geostatistics with Krige (1951) and Matheron's (1962) pioneering work, numerous new techniques have been developed to generate distributions of properties of interest at unsampled locations. Many of these techniques are based on the theory of kriging; introductory discussions of kriging can be found in geostatistical text books such as Journel and Huijbregts (1978), Deutsch (2002) or Isaaks and Srivastava (1989). One advantage of using kriging to generate estimates is the incorporation of anisotropy into the modeling process. Anisotropy is the concept that the properties in a geological deposit are often more continuous in one direction than another. Consider the locally varying anisotropy shown in these cross sections:

Figure 2: Cross sections displaying locally varying anisotropy. Left: Folding and faulting caused by the San Andreas Fault (www. strike-slip.geol.ucsb.edu). Right: Folding in the northern Rocky Mountains (www. mkutis.iweb.bsu.edu).

Anisotropy within geological formations can be exploited to increase the accuracy of modeling. If the direction and magnitude of anisotropy are well understood, they can be transferred into modeling to improve performance. Consider an LVA field modeled after the hand drawn directions on Figure 2 and two drill holes through the deposit. The problem is to estimate at all the unsampled locations. Techniques, such as inverse distance, that do not normally consider anisotropy cannot capture the horizontal continuity of the deposit (Figure 3 left). Other techniques, such as kriging, provide disappointing results because only a single direction of continuity can be incorporated into the modeling (Figure 3 middle). More geologically realistic results can be obtained by considering that the anisotropy varies locally (Figure 3 right); however, incorporating locally varying anisotropy in even this simple case is difficult. More complex anisotropy fields, such as Figure 2 left, are challenging.

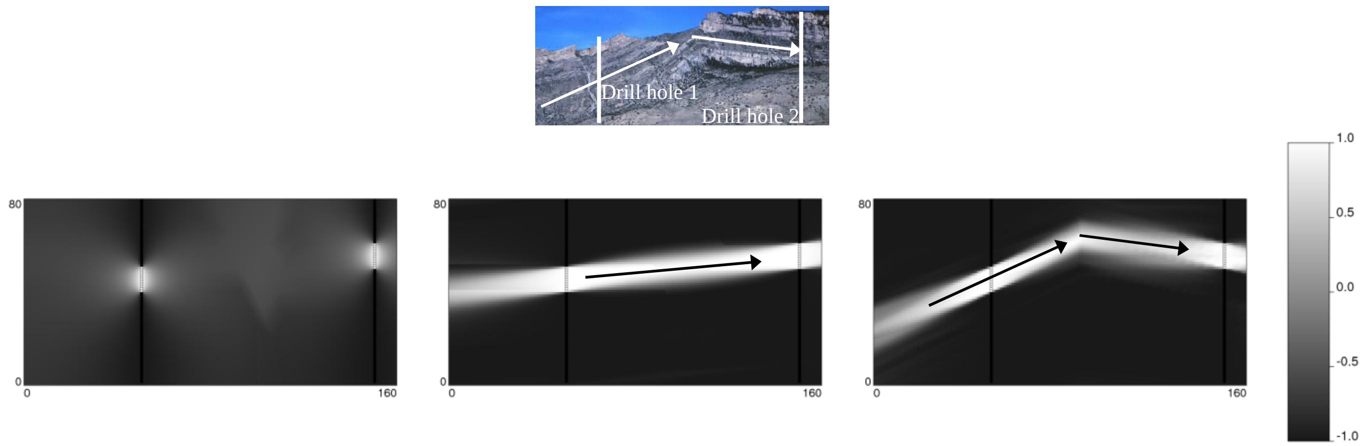

Figure 3: Top: cross section with the two drill holes (www.mkutis.iweb.bsu.edu). Lower Left: inverse distance estimation. Lower Middle: Kriging with horizontal anisotropy. Lower Right: Kriging considering locally varying anisotropy.

Method

Normally, in the case of a single direction of continuity, it is implicitly assumed that the straight line path between points is the path along which two points are related. Consider a simple folded sedimentary deposit (Figure 4). Points A and B are not related along a straight line path. Unfolding the deposit into its pre-fold state reveals the path along which points A and B are related. In the folded space, this path is not linear but follows the locally varying directions of continuity.

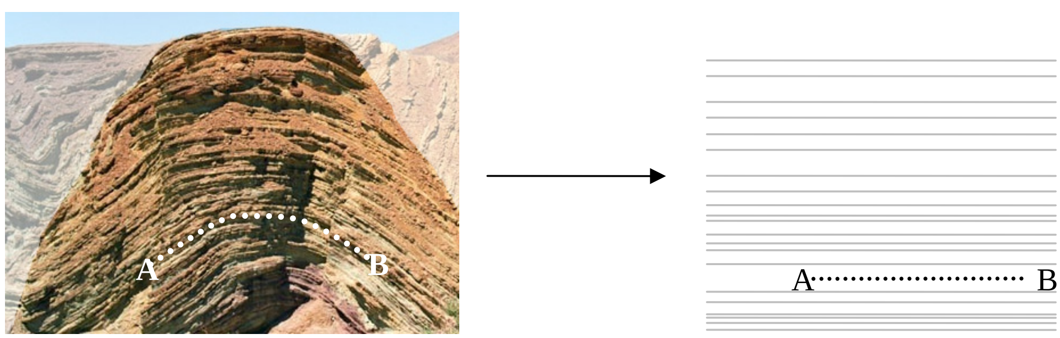

Figure 4: Left: An anticline with apparent locally varying anisotropy (www.geology.about.com). Points A and B are related to each other along a non-linear path because of the locally varying anisotropy. Right: Unfolding the anticline highlights the natural path between points A and B.

Consider a grid overlain on the anticline in Figure 4 with locally varying anisotropy shown in Figure 5. The anisotropy is now different in each cell of the model. Just as the anisotropic distance is different for a different anisotropy direction, each cell of the model (Figure 5) has a different direction of continuity and the anisotropic distance depends on the local direction of continuity. The distance between two points should be a sum of the distances within each cell as the distance is calculated differently if the anisotropy direction changes locally. For illustrative purposes consider the horizontal path between A and B. The anisotropic distance can be calculated in each of the 18 cells this path crosses and the distance from point A to B can be calculated as the sum. Considering the more realistic curved path between A and B intersects a total of 25 cells. Again the sum of the 25 individual distances represents the anisotropic distance between points A and B. Depending on the anisotropy field the curved path from A to B can be shorter than the straight-line path. This is the central concept used to incorporate locally varying anisotropy.

It is critical to note that this discussion is focused on the anisotropic distance not the Euclidian distance. In the case of constant anisotropy the shortest anisotropic distance is always found using the straight-line path; however this is not the case when considering locally varying anisotropy (Figure 5).

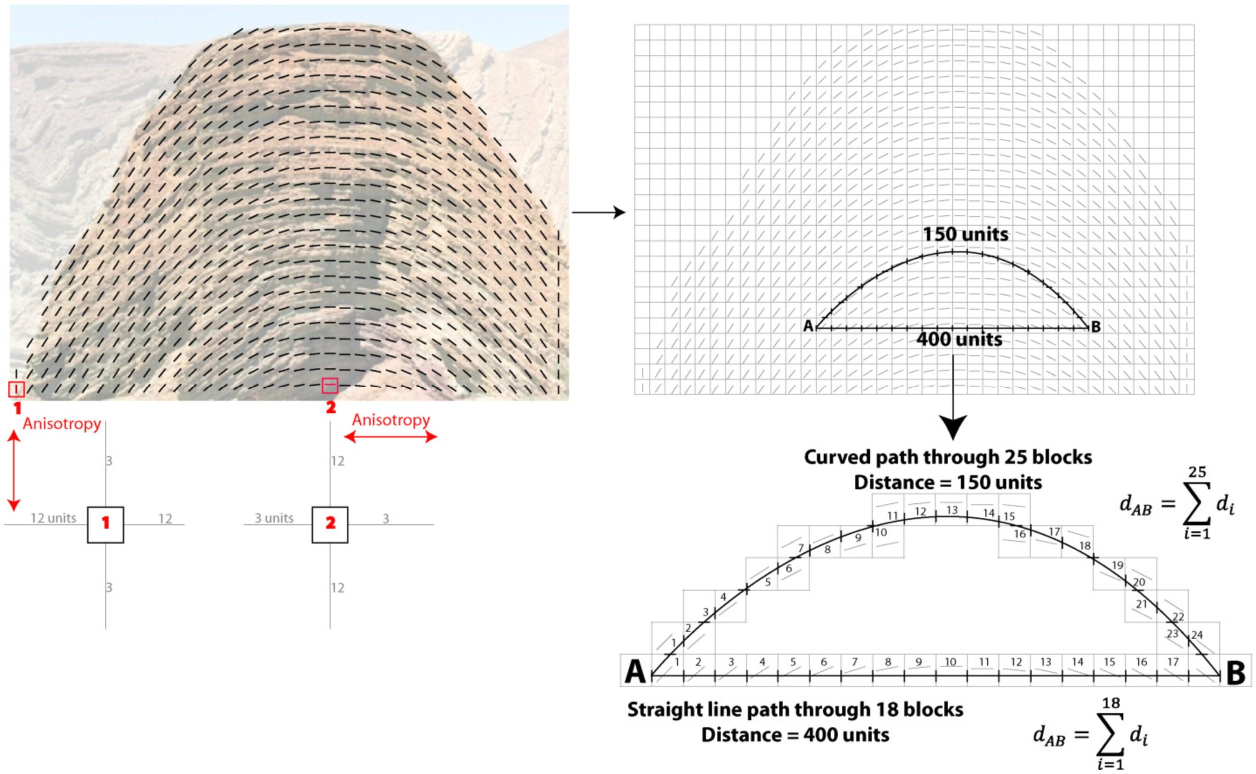

Figure 5: Left: An anticline with a locally varying anisotropy grid (www.geology.about.com). Locations 1 and 2 are highlighted to show how the anisotropic distance in each cell is different and depends on the locally varying anisotropy field. At point 1 a horizontal path would have an anisotropic distance of 3 units whereas a vertical path through the cell would have an anisotropic distance of 12 units. Right: two potential paths between points A and B, each path is the summation of the distance through each cell. The shortest distance between points A and B is 150 units.

The first step in this methodology is to calculate the distance between points using a non-linear path. Consider the grid as a graph, where the distance between nodes (or grid cells in a model) can be calculated using the underlying locally varying anisotropy:

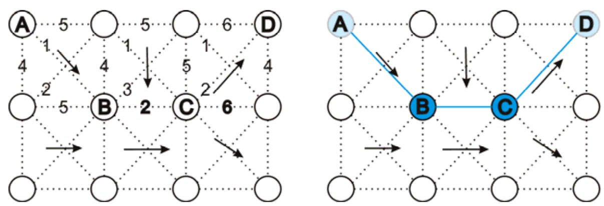

Figure 6: Left - Graph of a 2D grid, each cell is connected to adjacent cells and relevant distances are indicated on the graph. Right - Potential path between nodes A and D.   From Boisvert and Deutsch (2008).

The shortest path between points A and D (Figure 6) is not a straight line. Dijkstra's algorithm (Dijkstra, 1959) is used to calculate this path.

Using this non-linear path in kriging does not ensure positive definiteness of the resulting kriging equations. Multi Dimensional Scaling (MDS) is used to as a transformation of the coordinates to ensure positive definiteness. The reader is referred to Mardia, Kent and Bibby (1979) for a description of this technique.

The following two example highlight the strengths of considering this technique. In both examples, it would be difficult to incorporate the underlying features of the locally varying anisotropy grid (shown to the left of the kriged plots).

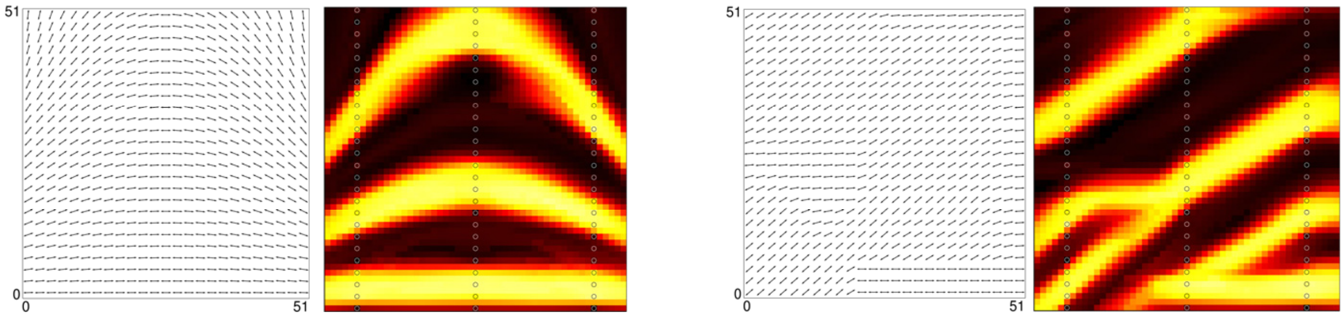

Figure 7: Left: A folded seam deposit with the associated kriging result and 3 drillholes (78 total samples). Right: A channel/vein deposit with the same 3 drillholes (78 total samples). From Boisvert and Deutsch (2008).

Conclusions

This paper explored the case when sufficient quantitative or qualitative knowledge is available to understand the locally varying geological features. It is assumed that the underlying anisotropy field is known. In such cases these features should be incorporated into geostatistical modeling. Locally varying anisotropy is incorporated by considering non-linear paths between points.

Increasingly practitioners are becoming concerned with incorporating non-stationarity into their models. The presented methodology can be computationally intensive as computer time is increased to days rather than hours for large models. This is the tradeoff for considering locally varying anisotropy in geo-modeling.

Acknowledgements

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