



Rock Physics Models for the Seismic Velocity of Cracked Media

Zimin Zhang*

University of Calgary, Calgary, Alberta

zzhan@ucalgary.ca

and

Robert R. Stewart

Abstract

Two petrophysical models for cracked media are investigated in this paper: the Kuster-Toksöz (1974) model for randomly oriented cracks and Hudson's (1981) model for aligned cracks. We consider the effects of crack shape, aspect ratio, and crack density using rock properties from several field locations: the Ross Lake heavy oil field, Saskatchewan; the Violet Grove, Alberta CO₂ injection site; and a Saskatchewan mining area. Generally, introducing cracks or inclusions into a rock can significantly decrease its P- and S-wave velocities. Inclusion shape has a large influence on the resultant rock properties from the Kuster-Toksöz model. Small aspect ratios (thinner cracks) can yield the largest decreases in velocities. Modeling results indicate that a 1% porosity, from penny-shaped cracks with an aspect ratio of 0.01, can produce up to 22% velocity decreases in Hudson's model. Similar inclusions create P-velocity decreases of 16% and S-velocity decreases of 11% in the Kuster-Toksöz model.

Introduction

The Kuster and Toksöz (1974) method calculates the isotropic effective moduli for randomly distributed inclusions in a rock based on a long-wavelength, first-order scattering theory. The Hudson (1981) model uses scattering of the mean wave field in an elastic solid with thin, penny-shaped cracks or inclusions which are aligned in a specific direction. The effective moduli can be calculated by applying first- and second-order corrections to the isotropic background moduli. The overall effect of the aligned cracks is anisotropic.

Both models assume no fluid flow between spaces, thus they simulate high-frequency, saturated-rock behavior. At low frequencies, when there is time for wave-induced pore pressure increments to flow and equilibrate, dry-rock moduli should first be calculated from the two models. Then Gassmann fluid substitution for isotropic media and Brown and Korringa's (1975) fluid substitution for anisotropic media could be used to predict saturated rock properties.

Rock properties for numerical test

Several rock types are selected to provide values for numerical tests: a Cretaceous-aged, high-porosity (about 30%) channel sand and a tight sand from the Ross Lake heavy oil field, another Cretaceous-aged low-porosity (about 12%) sandstone from Violet Grove, Alberta, and a Devonian carbonate and a shale from a potash mining area in Saskatchewan. These rock properties are listed in Table 1. The porous channel sand and tight sand from Ross Lake area were used for various parameter tests with the Kuster-Toksöz model and Hudson's model. For this modeling, the Hashin-Shtrikman bounds were also calculated for comparison. The Hashin-Shtrikman bounds are the narrowest constraints when the geometries of the constituents are not known. Cracked rock properties are also calculated for all the chosen rocks assuming penny-shaped cracks – with a fractional crack porosity of 0.01 and an aspect ratio 0.01. For all the tests, the void spaces are filled with brine at a density of 1.1 g/cm³ and velocity of 1430 m/s.

Kuster-Toksöz Model

Figures 1 and 2 display the results of randomly oriented inclusions in sands of the Ross Lake heavy oil field as calculated by the Kuster-Toksöz model. Dry moduli were calculated first by supposing that both bulk and shear modulus of the inclusions are 0, then the Gassmann equation was used to calculate the effective moduli when the void space is filled by brine. As shown in figure 1, we find that the velocities decrease significantly, depending on the inclusion shape. Smaller aspect ratio yields a larger decrease of velocities. The velocities of the sphere inclusion shape are coincident with the Hashin-Shtrikman upper bound. The effective velocities of the small aspect ratio shapes approach the Hashin-Shtrikman lower bound at a smaller volume fraction of pores. Except for the sphere shape inclusions, all other inclusion shapes have a limitation on volume fraction values for reasonable effective velocity values. The concentration value limitations decrease with aspect ratio. For needle shape inclusions, there is no dependence on aspect ratio. The results are valid for a large range of concentration values.

Table 1: Rock properties for numerical tests of cracked media.

Lithology	Ross Lake		Violet Grove	Sask. Mining	
	Sandstone	Sandstone	Sandstone	Carbonate	Shale
Depth	1148m	1160m	1605m	970m	1006m
Vp (m/s)	3026	5689	3778	5538	3765
Vs (m/s)	1721	3413	2237	2954	2074
Density (g/cc)	2.133	2.63	2.42	2695	2326
Porosity	30%	2%	12%	3%	<5%

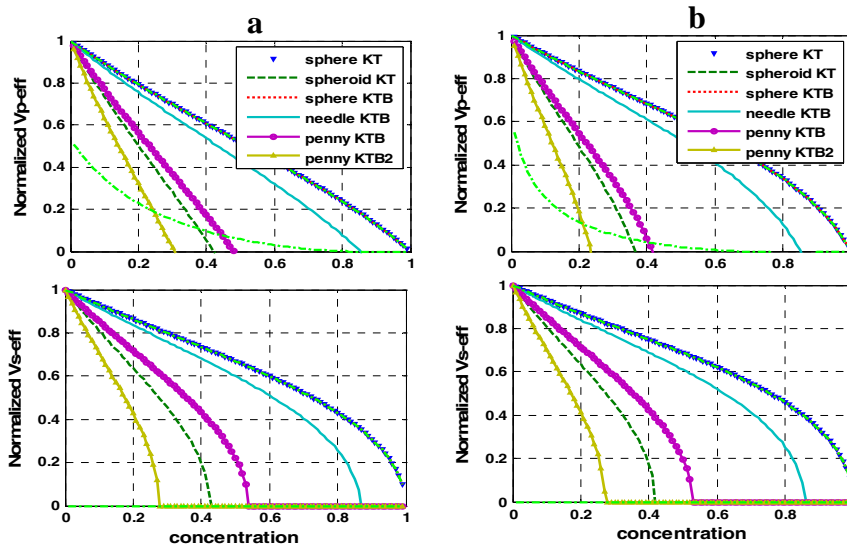


Figure 1. Variation of effective P- (top) and shear (bottom) velocities with the volume concentration of inclusions for several crack shapes from the Kuster-Toksöz model (a: Ross Lake porous channel sand; b: Ross Lake tight sand). All the velocity values are normalized to the range from fluid to uncracked rock velocities. The aspect ratio value for the oblate spheroid shape is 0.1. For the penny shapes, an aspect ratio of 0.1 (noted as penny KTB) and 0.05 (noted as penny KTB2) are used. KT: the results from the Kuster-Toksöz formula for sphere and oblate-spheroid inclusions; KTB: the results from generalized Kuster-Toksöz model by Berryman. The green dash-dot lines are Hashin-Shtrikman bounds.

A test of the a/c value with different c values (0.01, 0.05, and 0.25) was also carried out and the results are shown in Figure 2. The velocity values were normalized by the Hashin-Shtrikman upper and lower bound. For various c values, both P- and S-velocity indicate relatively stable minimum a/c values, approximately 0.2 (0.4 from moduli calculation results). However, for penny-shaped inclusions, the maximum a/c values for reasonable velocities change drastically with respect to the crack concentration value c . Small c values will still have reasonable effective velocities for large a/c values. The P-velocities have less limitation on the a/c values than S-velocities. For spheroid inclusions, there is no upper limitation on the a/c value, but with increasing c value, the effective velocities approach upper bound quickly.

Hudson's model

Figure 3 shows the results from Hudson's model for penny cracks with three aspect ratios (α): 0.002, 0.01, and 0.05. It displays the modeled P- and S-velocity variations with crack density. For the cracks aligned in one direction, it will show transverse anisotropy with respect to the axis along the normal to the cracks. The P-velocity drops very little when the waves travel along crack plane (V_{p0}), but will display a distinct

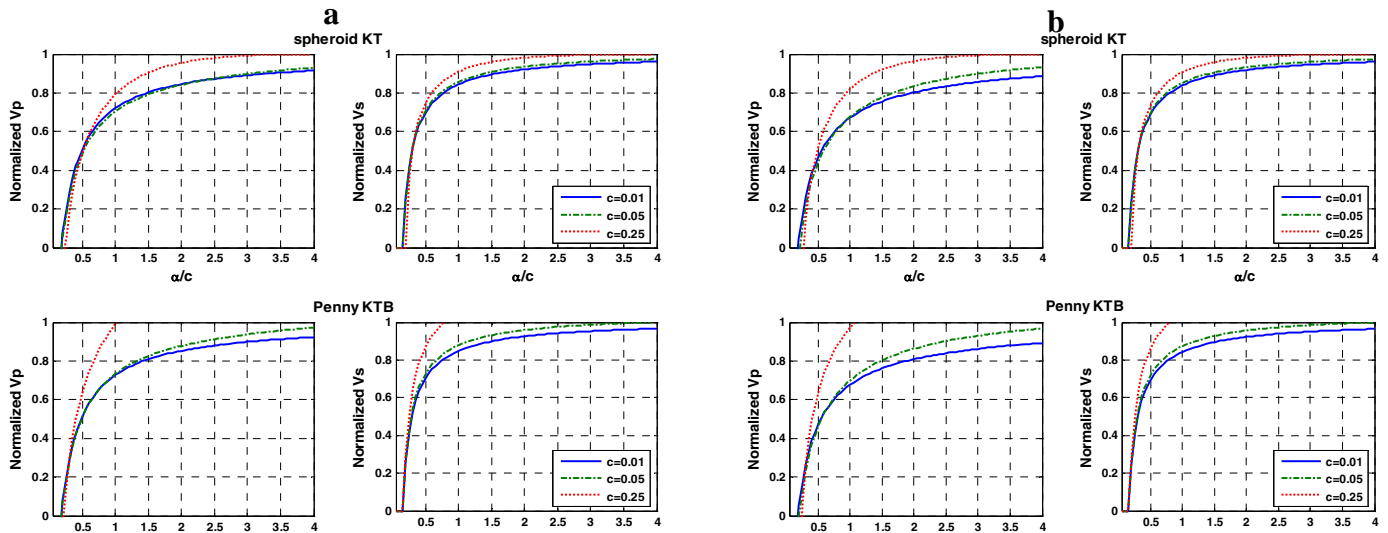


Figure 2: Variation of effective P- (top) and shear (bottom) velocities (from the Kuster-Toksöz model) with a/c value (aspect ratio/volume concentration) (a: Ross Lake porous channel sand; b: Ross Lake tight sand). All the values are normalized to the range of Hashin-Shtrikman bounds.

decrease when the wave travels normally to the cracks (V_{p90}). For SV waves, the velocity will change the same amount whether it travels normal to the cracks or across the crack plane. Cracks with aspect ratio 0.05 were also modeled by the Kuster-Toksöz model for penny-shaped cracks. The effective P velocities from the Kuster-Toksöz model are between the P velocities from Hudson's model along the crack normal and crack plane. For given aspect ratio cracks, when the crack density exceeds a limit, the velocities will display an abnormal increase with crack density value, especially for V_s . This is about 0.05 (a 0.1% crack porosity equivalent) for cracks with aspect ratio 0.002 and 0.2 (around 1% crack porosity) for cracks with aspect ratio 0.01. From the modeling results for tight sand from Ross Lake (Figure 3b), we find that the P-velocity variations with crack density show an apparent dependence on rock properties of the uncracked rock. While the S-velocity displays a similar variation with crack density. However, reasonable crack density ranges for each aspect ratio are still the same due to the similar variation of S-velocity with crack density.

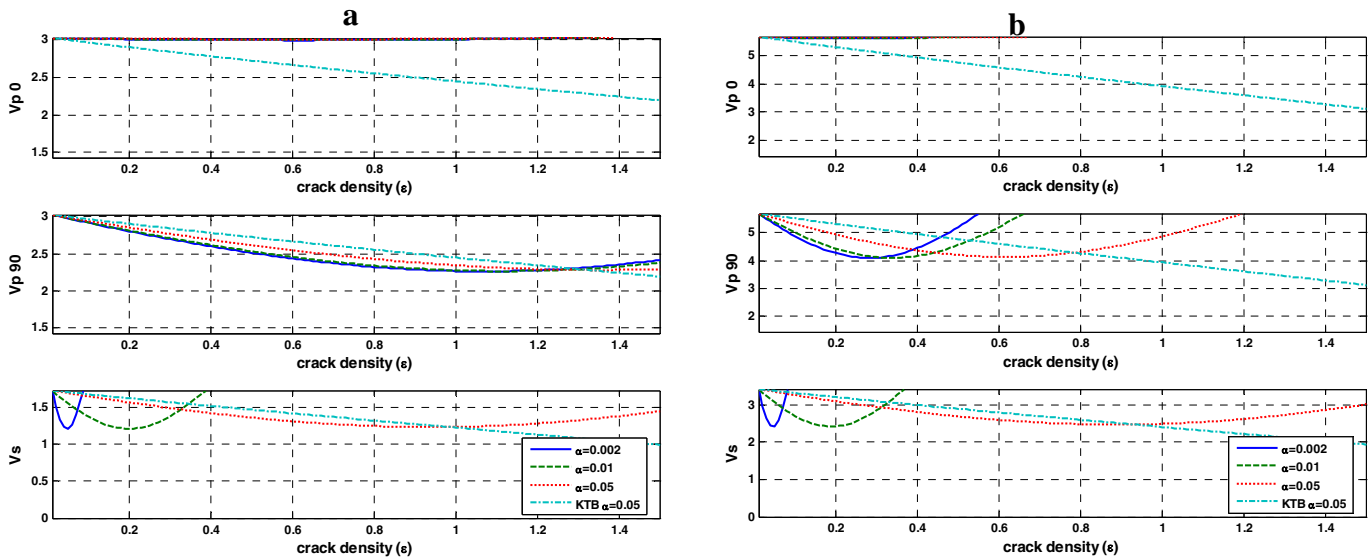


Figure 3: Variation of effective velocities of cracked rock from Hudson's model with crack density (V_{p0} : V_p along crack plane; V_{p90} : V_p normally to the cracks). a: Ross Lake porous channel sand; b: Ross Lake tight sand. The minimum and maximum velocity values in the plot are those of fluid and isotropic uncracked rock respectively. KTB denotes the effective velocities from Kuster-Toksöz model.

Assuming a 1% crack porosity induced by penny-shaped cracks with aspect ratio 0.01, the effective P- and S-velocities from Kuster-Toksöz and Hudson's model are plotted in Figure 4 for: 1) the Ross Lake porous

sand, 2) Saskatchewan mining shale, 3) Violet Grove sand, 4) Saskatchewan mining carbonate, and 5) Ross Lake tight sand. We find:

1. These cracks can produce up to 22% velocity decreases in Hudson's model, and P-velocity decreases of 16% and S-velocity decreases of 11% in the Kuster-Toksöz model;
2. The percentage changes of S-velocity induced by the cracks are quite similar for each rock from both models;
3. The percentage changes of P-velocity along crack planes are very similar from Hudson's method without or with fluid substitution;
4. The percentage changes of P-velocity (P-velocity along the crack normal for Hudson's model results) are consistent with the values of uncracked rocks from Kuster-Toksöz model and Hudson's method without fluid substitution.

Conclusions

Two petrophysical models (from Kuster-Toksöz and Hudson) for cracked media are discussed. From the results of the Kuster-Toksöz model, we find that the changes in rock properties depend largely on the inclusion shape. For both spheroid and penny shape inclusions, a/c values should not be smaller than about 0.4 (equivalent to $c < 2.5a$). As for penny shape inclusions, the valid maximum a/c values change drastically with respect to the concentration value c . For Hudson's model, smaller aspect ratio cracks have a smaller valid crack density range, especially for V_s .

The modeling results for several rocks (assuming 1% crack porosity, 0.01 aspect ratio penny-shaped cracks) indicate: these cracks can produce up to 22% velocity decreases in Hudson's model, and P-velocity decreases of 16% and S-velocity decreases of 11% in the Kuster-Toksöz model; the percentage changes of S-velocity from both models and P-velocity along crack planes from Hudson's method have almost no dependence on uncracked rock properties; while the percentage changes of P-velocity (P-velocity along crack normal for Hudson's model results) are consistent with the values of uncracked rocks for Kuster-Toksöz model and Hudson's method without fluid substitution.

Acknowledgements

Support by CREWES sponsors is gratefully appreciated.

References

- Berryman, J.G., 1980. Long-wavelength propagation in composite elastic media. *J. Acoust. Soc. Am.*, **68**, 1809-1831.
- Brown, R., and Korrington, J., 1975. On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid. *Geophysics*, **40**, 606-616.
- Hudson, J.A., 1981. Wave speeds and attenuation of elastic waves in material containing cracks. *Geophys. J. Royal Astronom. Soc.* **64**, 133-150.
- Kuster, G.T., and Toksöz, M.N., 1974, Velocity and attenuation of seismic waves in two-phase media: Part I. theoretical formulations. *Geophysics*, **39**, 587-606.

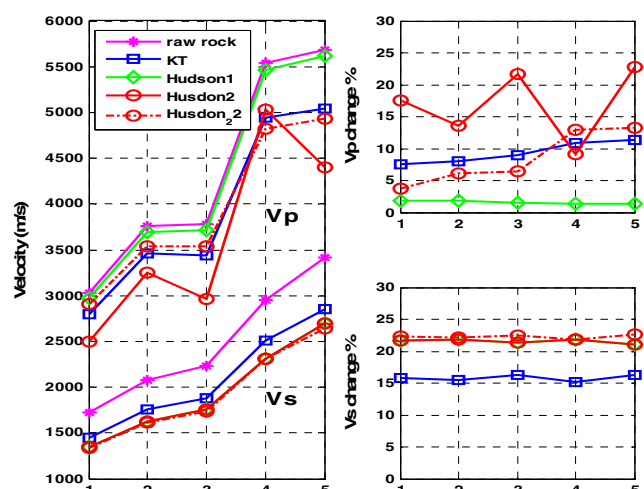


Figure 4: Modeled effective P- and shear velocities for selected reservoir rocks (rock samples 1 through 5) assuming penny shape cracks with aspect ratios 0.01 and a crack density 0.01. KT: velocities from Kuster-Toksöz model. Hudson 1: velocities along the crack plane; Hudson 2: velocities along crack normal; Hudson₂: velocities along crack normal without fluid substitution. The plots in the right are percentage changes with respect to the original velocity.