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The Phase Shift Time Stepping Equation and the Marmousi Data Set

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Summary

As a result of the numerical performance of finite-difference operators, reverse-time migration (RTM) produces images which are typically low frequency or require large computational resources. We consider an alternative to wavefield propagation with finite differences, a two-way high-fidelity time-stepping equation based on a windowed Fourier transform which is exact for constant velocity medium.

Introduction

Reverse-time migration (Baysal et. al., 1983, McMechan, 1983; Whitmore, 1983) is a depth migration algorithm. It is capable of imaging reflectors using overturned waves and multiples. However, as a result of the sampling requirements, processing seismic surveys will either require harsh filtering to remove higher frequency data, or they will require long run times even with a cluster of computers. The fine sampling requirements occur because finite-difference operators propagate high frequencies with an incorrect dispersion relation. Pseudospectral methods compute the Laplacian in the Fourier domain which is an infinite order approximation in space but uses a finite difference method for the time derivative and so still suffers from numerical dispersion. We propose an alternative explicit time-stepping equation that does not use finite differences. An analytical solution to the acoustic wave equation (Etgen, 1989; Tal-Ezer, 1986) is computed exactly in the Fourier domain and so do not suffer from dispersion at high frequencies. As a result, the sampling requirements are better than propagating with finite differences. Similarly to phase-shift depth stepping methods (e.g. Gazdag and Sguazzero, 1984) and Gabor depth stepping (Ma and Margrave, 2008), we extend our method to variable velocity by replacing the global Fourier transform with a local Gabor transform using localizing windows within each of which a locally homogeneous solution is computed.

The Phase-Shift Time Stepping Equation

The Fourier transform over the spatial dimensions of the constant-velocity scalar wave equation is

$$(2p)^2\left(k_x^2 + k_z^2\right)\hat{U} + \frac{1}{c^2}\hat{U}_{tt} = 0, \quad (1)$$

where $\hat{U}$ is the wavefield in the spatial Fourier domain, $\hat{U}_{tt}$ is the second time derivative of the $\hat{U}$, x is the lateral coordinate, z is the depth coordinate, t is the time coordinate, c is the speed of propagation, and k_x, k_z are the wavenumbers which correspond to the coordinates x, z, respectively. Subject to the initial conditions $U(t,x,z) = U_t(x,z)$, and $U\left(t - Dt, x, z\right) = U_{t-Dt}\left(x,z\right)$. The resulting exact solution of the constant-velocity wave equation is

$$U(t+\Delta t,x,z) = -U(t-\Delta t,x,z) + \int \hat{U}(t,k_x,k_z)\cos\left(2\pi c \|\vec{k}\| \Delta t\right) e^{2\pi i \vec{x}\cdot\vec{k}} dk_x dk_z, \quad (2)$$

where $\vec{x} \cdot \vec{k}$ is the scalar product of the 2D position and wavenumber vectors. This equation may be used recursively to compute a solution to the wave equation. The size of the timestep is limited by an aliasing condition $r = \Delta t c / \Delta x < 1/\sqrt{2}$. Equation (2) may be adapted to variable velocity by replacing the constant velocity c with a variable velocity $v(x,z)$. The resulting approximate solution to the variable velocity wave equation is too numerically complex to be used directly for RTM. We use a Gabor windowing scheme to approximate it. Suppose that the velocity $v : R^2 \to R$ and $I_j : R^2 \to \{0,1\}$, $j \in J$ such that $\sum_{j\in J} I_j = 1$ and $\| \sum_{j\in J} v_j I_j - v(x,z) \| < \varepsilon$ for any positive ε where R denotes the real numbers. The I_j are spatially discontinuous "windows" that are unity wherever the spatially constant v_j is sufficiently close to v. Suppose that $w : R^2 \to R, \|w\|_{L^2} = 1$ is smooth function localized at $\vec{x} = 0$. To prevent internal reflections the I_j windows are smoothed, $\Omega_j = w * I_j$, where * denotes 2D spatial convolution. The partition W_j is used to window the wavefield into regions with approximate constant velocity. Each region W_j is then propagated with the constant velocity v_j .The partition is used to window the wavefield into regions at each time step and the combination of windowing and Fourier transformation converts equation (2) into

$$U(t+\Delta t,\vec{x}) = -U(t-\Delta t,\vec{x}) + \sum_{j\in J} \iint \cos\left(2\pi v_j \|\vec{k}\| \Delta t\right) \hat{U}\left(t,\vec{k}\right) e^{2\pi i (\vec{x}-\vec{y})\cdot\vec{k}} W_j(\vec{y}) dy_1 dy_2 dk_1 dk_2, \quad (3)$$

where $\vec{y} = (y_1, y_2)$ and $\vec{k} = (k_1, k_2)$.

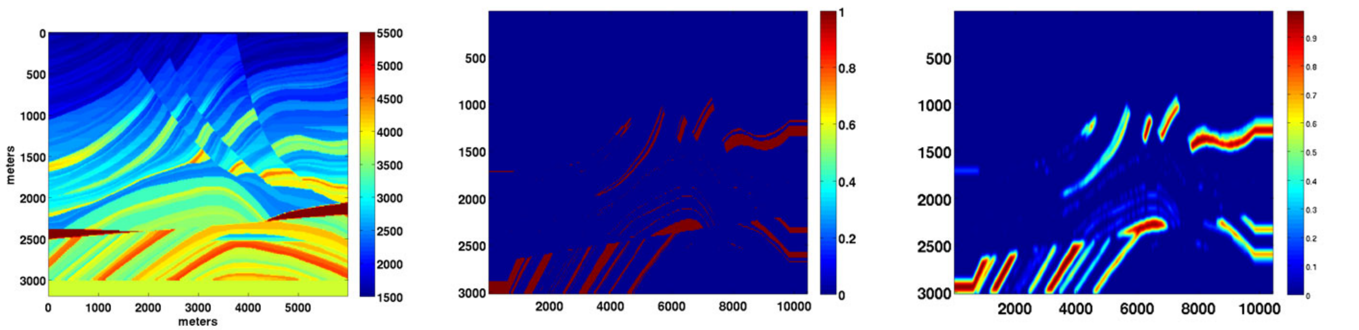

Figure 1: Left:Velocity model for Marmousi data set (Versteeg, 1994). Center: An I_j velocity partition. Right: An Ω_j velocity partition.

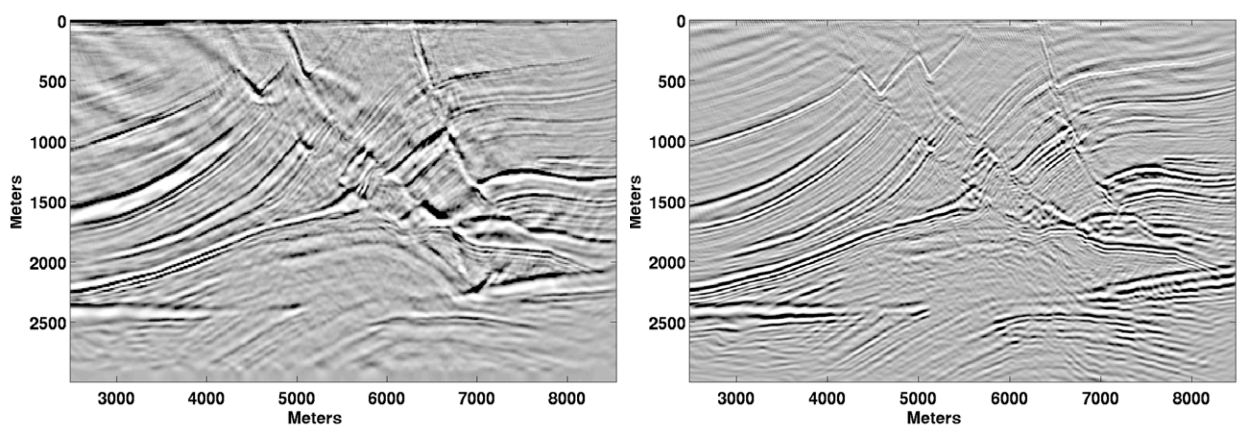

Figure 2: Left: An image the Marmousi data set using PSTS RTM. Right: The run time for migrating a shot record is 12 minutes. An error criteria was used (40m/s) which produced 18 reference velocities. An image of the Marmousi data set using 2nd order time and 4th order space explicit finite difference RTM. The run time for migrating a single shot record with FD is 24 minutes.

To benchmark PSTS RTM we compare it to a 2nd order time and 4th order space finite-difference RTM. Figure 2 is a comparison of the Migrated Marmousi data set using PSTS equation (right) and finite differencing the wave equation (left). The FD RTM appears to be of higher quality. This is under investigation and may be due to artifacts associated with lower sample rate in the imaging condition. The FD image took twice as much time to calculate.

Phase-Shifting with the Derivative of the Wavefield

Solving the wave equation requires two initial conditions. As an alterative to equation (2), we derive a two way phase-shifting time stepping equation using different initial conditions. This equation propagates the derivative of the wavefield and the wavefield forward in time. In doing so it does not suffer from an aliasing restriction on the timestep but requires twice as much computation and memory as equation (2) to compute one timestep. The timestep is limited by the need to window in the space domain which allows propagation with the local velocity and by the need to calculate the imaging condition at each spatial position of the waves. Let $V(t,\vec{x}) = \partial U(t,\vec{x}) / \partial t$ be the derivative of the wavefield. Define the matrix kernel

$$K_j\left(v_j, \|\vec{k}\|\right) = \begin{bmatrix} \cos\left(2\pi v_j \|\vec{k}\| \Delta t\right) & \dfrac{\sin\left(2\pi v_j \|\vec{k}\| \Delta t\right)}{2\pi v_j \|\vec{k}\|} \\ 2\pi v_j \|\vec{k}\| \sin\left(2\pi v_j \|\vec{k}\| \Delta t\right) & \cos\left(2\pi v_j \|\vec{k}\| \Delta t\right) \end{bmatrix}. \quad (4)$$

Adapting the solution to the constant velocity solution to the wave equation (Stein,1993) by a Gabor decomposition of the wavefield,

$$\begin{bmatrix} \hat{U}(t+\Delta t,\vec{k}) \\ \hat{V}(t+\Delta t,\vec{k}) \end{bmatrix} = \sum_{j \in J} K_j\left(v_j, \|\vec{k}\|\right) \begin{bmatrix} FT\left\{W_j(\vec{x}) U(t,\vec{x})\right\} \\ FT\left\{W_j(\vec{x}) V(t,\vec{x})\right\} \end{bmatrix}, \quad (5)$$

where FT are denotes the forward Fourier transform over the spatial coordinates $\vec{x}$. Figure 3 compares three snapshots propagated through the Marmousi dataset. The snapshots computed with equation (5) were computed with a much larger timestep that the snap shot computed with equation (3).

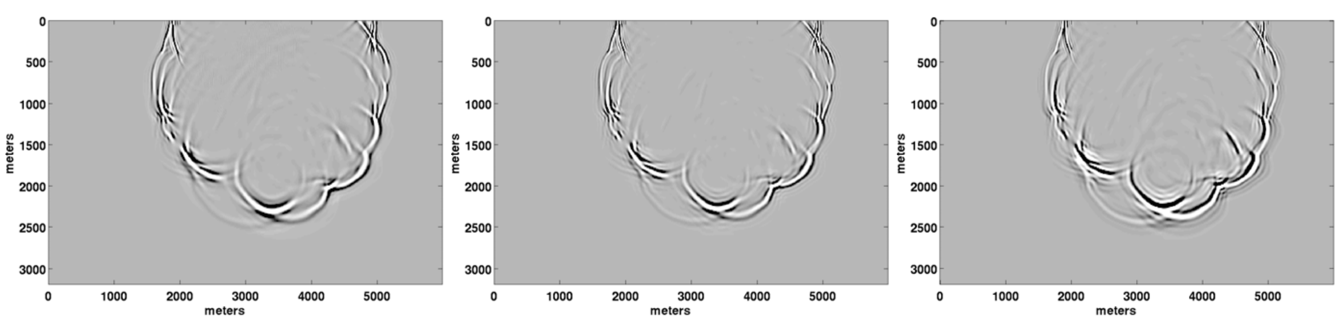

Figure 3:: Snap shot of the wavefield through the Marmousi data set using equation (5) with time sample rate of 8ms (left) and 4ms (center). Snapshot of wavefield through the Marmousi data set using equation (3) with timesample rate of 1.5ms (right).

A Linear Velocity Approximation

To reduce the number of reference velocities we derive a phase shifting equation that allows the use of the FFT for a medium velocity with a constant gradient. A Taylor series is used to approximate the cosine operator in the variable z about the point z_0. The 2nd order power series expansion about the point $z = z_0$ for the cosine function is

$$\cos\left(2\pi\left(v + a(z - z_0)\right)\|\vec{k}\|\Delta t\right) \approx \cos\left(2\pi v_0\|\vec{k}\|\Delta t\right) + \sin\left(2\pi v_0\|\vec{k}\|\Delta t\right)a(z - z_0)2\pi v_0\|\vec{k}\|\Delta t \;. \quad (6)$$

Figure 3 compares using equation (6) using 4 reference velocity windows to migrating with equation (3) with 13 reference velocities.

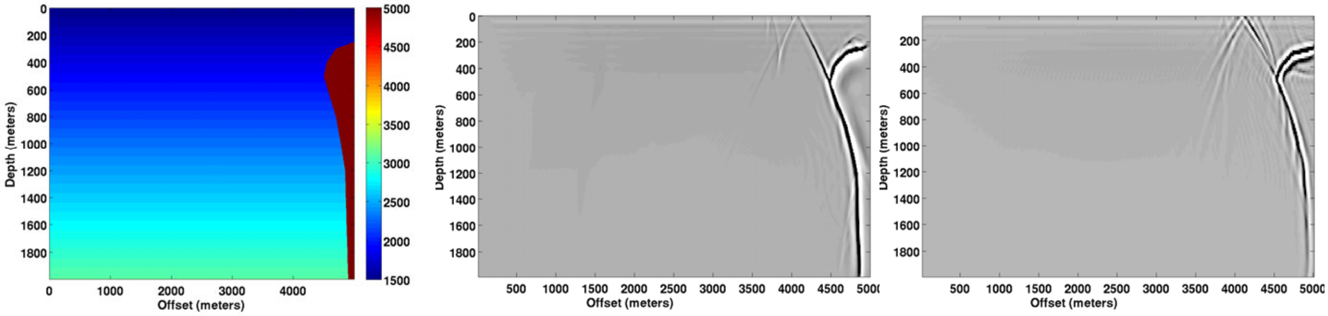

Figure 3: Left: Model of an overhung saltdome. Middle: Poststack migration using 3 linear velocity windows. Right: Poststack migration of saltdome using 13 constant velocity windows.

A modelling Example

The PSTS equation can used to image multiples. To demonstrate this we use equation (3) as a modelling algorithm. The velocity windows Ω_n are not smoothed to suppress internal reverberations. Figure 4 compares two shot records. The shot record using finite differences contains some numerical dispersion but attenuates the reflections off the boundaries better.

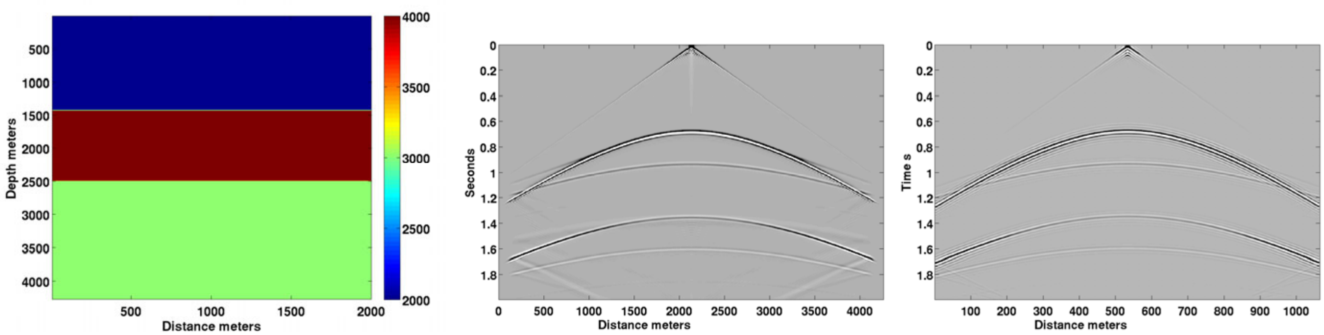

Figure 4: From the left to right. A velocity model with sharp constrasts. A shot record using generated using equation (2). Shot record generated by 2nd order time 4th order space finite differeces.

Conclusions

We have used the two-way PSTS equation to migration a saltdome and the Marmousi data with promising results. We also used it to successfully forward model a shot record.

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