Joint PP and PS AVO Inversion Based on Zeopprtiz Equations

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Summary

This paper makes efforts to explore joint analyzing the amplitude versus offset phenomena in the PP and PS data with expectation to reduce the ambiguity of AVO analysis by utilizing the redundancy of multi-component AVO measurements. The method uses the exact Zoeppritz equations to inverts for the ratio of density, P-wave velocity and S-wave velocity using the PP and PS plane wave reflection coefficients. At small incident angle ($<30^o$), linear AVO response of PP and PS only can provides two independent attributes respectively (gradient and accept for PP reflectivity, pseudo gradient and pseudo accept for PS reflectivity). The power function fitting presented provides a higher precision AVO attributes than traditional polynomial fitting. By using four independent fitting attributes (two independent attributes for PP and PS respectively), the inversion of four ratio parameters (velocities and densities) would be estimated with less errors than that in traditional method.

Introduction

AVO analysis and inversion techniques have been used throughout the world in a variety of situations with varied rates of success. In the attempt to understand subsurface lithology, it is useful to jointly apply the properties of PP and PS AVO. Amplitude-versus-offset (AVO) analysis tries to infer S-wave velocities (or Poisson's ratio) from the change of P-wave reflectivity R_{pp} with varying angles of incidence. The change in R_{pp} is partially controlled by the coversion of P-wave into S-wave energy, according to the S-wave velocities. On the other hand, converted-wave (P-to-S) reflectivity is generally more dependent on the S-wave velocity. So if our goal is to find S-wave properties, it is reasonable to try to use converted-wave reflectivity R_{ps}. A number of authors have demonstrated the utility of combined AVO analysis using P-P and P-S (Miles and Gassaway, 1989). In an effort to constrain some of the problems inherent in standard weighted stacking schemes, Stewart (1990) developed a joint P-P and P-S weighted stacking technique proposed by Smith and Gidlow (1987) for PP AVO inversion. This method while maintaining the robustness of the P-wave weighted stacking technique has the benefit of data redundancy provided by the converted wave data. One problem inherent with the Smith and Gidlow approach was the incorporation of Gardner's relation, which may or may not be applicable in a given lithology. This method was further developed by Fatti et al. (1994) to eliminate this dependency upon Gardner's relation to estimate density. Recent work has shown that a Zeoppritz-based AVO inversion can improve the accuracy for elastic parameters estimates (Larsen,1999).

Joint PP and PS inversion

The Zoeppritz equations fully describe the relationship between incident, reflected and transmitted P and S plane waves on either side of a plane interface. The equations are algebraically quite complex and are difficult to inversion, Instead, a lot of useful linear approximations are presented. According to two parameters PP and PS linear approximations, it is found that there are only two independent AVO attributes for PP and PS reflection coefficient respectively. So stand-alone PP and PS AVO inversions are only suitable to invert two parameters steadily, joint PP and PS AVO inversion is suitable to invert four parameters steadily. The linearization (or approximation) of the Zoeppritz equations introduces error. In addition, the uniqueness of a solution to a given inverse problem is dependent upon the selection of the parameters to be inverted. Since the Zoeppritz equations are nonlinear functions with respect to the six elastic parameters, such as $\alpha_1, \alpha_2, \beta_1, \beta_2, \rho_1, \rho_2$, The ultimate goal of an AVO inversion method is to find the elastic parameters input to the Zoeppritz equations. To attain this goal in all settings, it is necessary to use non-linear inversion methods. A number of authors have discussed the use of these non-linear inversion methods for pre-stack P-wave seismic data. The goal in this study is to apply these P-wave methods to a simultaneous P-P and P-S non-linear inversion technique. The application of non-linear methods is more difficult due to the non-linear dependence between elastic parameter contrasts and reflectivity compared with the linearized case. Generally, the invertable parameters (the model parameters vector) are chosen $P = [\alpha_1, \alpha_2, \beta_1, \beta_2, \rho_1, \rho_2]$ (Larsen,1999).As we all know, reflection coefficient of PP and PS is the integrated influence of elastic parameters across interface, and the model which elastic parameters are multiplied by a constant will make the same response as original model. Therefore, the absolute value of model parameters is available without strict constraint.

We reduce the invertable model parameters to four $P = [\rho_2/\rho_1, \alpha_2/\alpha_1, \beta_2/\alpha_1, \beta_2/\alpha_1]$ .Following least square error theory, the object function of nonlinear inversion of the Zeoppritz equations is residual error function. For the case of joint of PP and PS inversion, the object function is written as following:

$$\varepsilon_{pp,ps}(P) = \sum_{i=1}^{N} [S_{pp,i} - M_{pp,i}(P)]^2 + \sum_{i=1}^{N} [S_{ps,i} - M_{ps,i}(P)]^2 \qquad (1)$$

where $M_{pp}(P)$ and $M_{pp}(P)$ are the forward model response for forward model response for a given set of model parameters vector, i.e. the PP and PS reflection coefficient Zoeppritz equation solution respectively. S_{pp} and S_{ps} are the real seismic record information (observations) of PP and PS respectively. If we select reflection amplitude of multi-offset as observations, the S and M denote the reflection coefficient. If we select AVO attributes, the S and M denote AVO attributes.

For angles of incidence up to 30 and small contrasts in elastic properties, P-P and PS AVO equation can be simplified to a linearized approximations with only two coefficients(Ramos and Castagna,2001), which only satisfies the inversion precision at small incidence angles at which the signal to noise ratio is lower for converted waves. At middle to far offset the converted wave amplitude reaches an extreme and the signal to noise ratio is higher.

The two term linearized approximations (Ramos,2001) of PP and PS can be abstract to

$$y = a_{pp}x^2 + c_{pp} \quad \text{(PP wave)} \qquad y = a_{ps}x^3 + c_{ps}x \quad \text{(PS wave)} \qquad (2)$$

Due to the linearized approximations of PP and PS is first order equation of Taylor series, so equation 4 can be generalized to:

$$y = a_{pp}x^{b_{pp}} + c_{pp} \quad \text{(PP wave)} \qquad y = a_{ps}x^{b_{ps}} + c_{ps}x \quad \text{(PS wave)} \qquad (3)$$

where coefficient b is variable, so equation 3 includes effects caused by higher terms and improves the fitting precision at middle and far offset.

Examples

Table 1 list a shale/gas model elastic parameters (Ramos,2001).Model parameters vector is $P = [0.9345\ 1.1119\ 0.5487\ 0.8448]$ .Figure 1 shows the PP and PS AVO responses to model. PP AVO response is almost linear at small incident angle (<30^o), whereas the PS AVO response reach an extreme at middle incident angel. Four incidence angles (5,10,15 and 20) are chosen for PP inversion alone and four incident angles (10,25,35 and 45) are chosen for PS inversion alone. Specially two PP data (such as 10 and 20) and two PS data (such as 35 and 45) are chosen for joint PP and PS inversion. To test the convergence of object function around a large neighborhood of chosen model parameters, given parameters ρ_2/ρ_1 and α_2/α_1 are the value of model parameter 0.9345 and 1.1119, parameter β_1/α_1 ranges from 1.1 to 1.5 and β_2/α_1 ranges from 0.7 to 2.8. The results are show in figure 2. The counter of PP and PS alone inversion has multi-extreme, moreover closed contours of joint PP and PS inversion shows a single minima. Nonlinear inversion of PP and PS alone easily fall into local solution and the joint inversion of PP and PS anyway search for optimized solution (Table 2). At small incident angle (<30^o), linear AVO response of PP and PS only can provides two independent attributes respectively (gradient and accept for PP reflectivity, pseudo gradient and pseudo accept for PS reflectivity). At middle offset the converted wave amplitude reaches an extreme and has high S/N ratio. Converted wave AVO curve fits using the power function presented here has a higher precision than third-order polynomial fits (Wei,2008), so power function fits is expanded to fit PP and PS AVO amplitude at middle offset and then provides four high precision independent attributes for joint AVO inversion. Figure 2 shows the fractional error of model parameters (Ramos,2001) obtained from joint PP and PS inversion. The inversion accuracy is significant better.

Conclusions

This paper has presented a practical method for joint inversion of PP and PS reflection data, which modified Zeoppritz equations as functions of four model parameters $P = [\rho_2/\rho_1, \alpha_2/\alpha_1, \beta_2/\alpha_1, \beta_2/\alpha_1]$. Next a nonlinear inversion method was constructed to simultaneously invert the exact Zoeppritz equation solution for PP and PS reflectivity. The object function of PP and PS alone inversion has multi-extreme and it is difficult to converge to true model parameters. Whereas, the object function of joint PP and PS inversion shows a single minima around a large neighborhood of true model parameters, so it is easy to search the global optimization solution. The power function fitting presented provides a higher precision AVO attributes than traditional polynomial fitting. By using four independent fitting attributes (two independent attributes for PP and PS respectively), this inversion promises better estimates to elastic parameters ratio

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References

Fatti, J.L., Smith, G.C., Vail, P.J., Strauss, P.J. and Levitt, P.R., 1994, Detection of gas in sandstone reservoirs using AVO analysis: A 3-D seismic case history using the Geostack technique: Geophysics, 59, 1362-1376.

Larsen J.A,1999, AVO Inversion by Simultaneous P-P and P-S Inversion, M.Sc. Thesis, University of CalgaryDepartment of Geology and Geophysics

Miles, D.R., and Gassaway, G.S., 1989, Three-component AVO analysis: 59th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 706-708

Ramos,A.B.C, and Castagna, J.P.,2001, Useful approximations for converted-wave AVO. Geophysics,2001,66(6):1721-1734

Stewart, R.R., 1990, Joint P and P-SV Inversion: The CREWES Research Report, Vol. 3, 112-115.

Smith, G.C., and Gidlow, P.M., 1987, Weighted stacking for rock property estimation and detection of gas: Geophysical Prospecting 35, 993-1014.

Wei X.C. Li X.Y. et al. 2008, Converted wave AVO inversion for average velocity ratio and shear wave reflection coefficient, 78th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 269-272

Table 1 model parameters

	Shale	Gas sand
α (km/s)	2.7700	3.0800
β (km/s)	1.5200	2.3400
ρ (g/cm^3)	2.2900	2.1400

Table 2 inversion of model parameters vector

Parameters	ρ_2/ρ_1	α_2/α_1	β_1/α_1	β_2/α_1
Exact	0.9345	1.1119	0.5487	0.8448
PP	0.6538	1.5906	0.8636	1.3304
PS	0.4018	0.2421	0.6585	1.2467
PP+PS	0.9345	1.1119	0.5487	0.8447

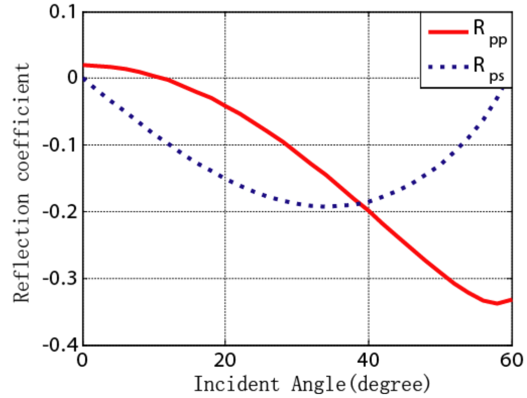
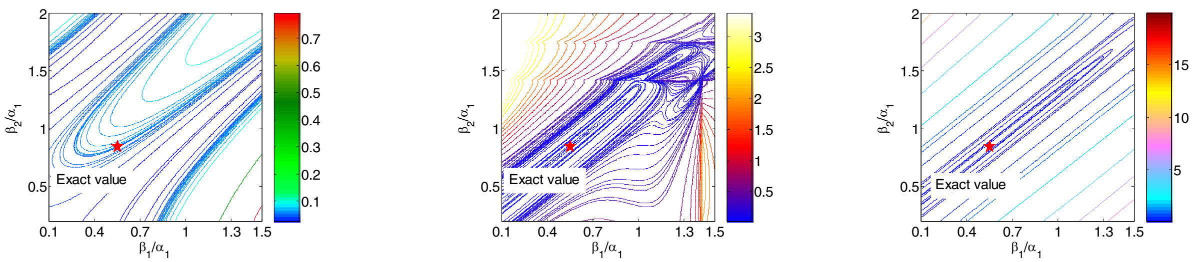
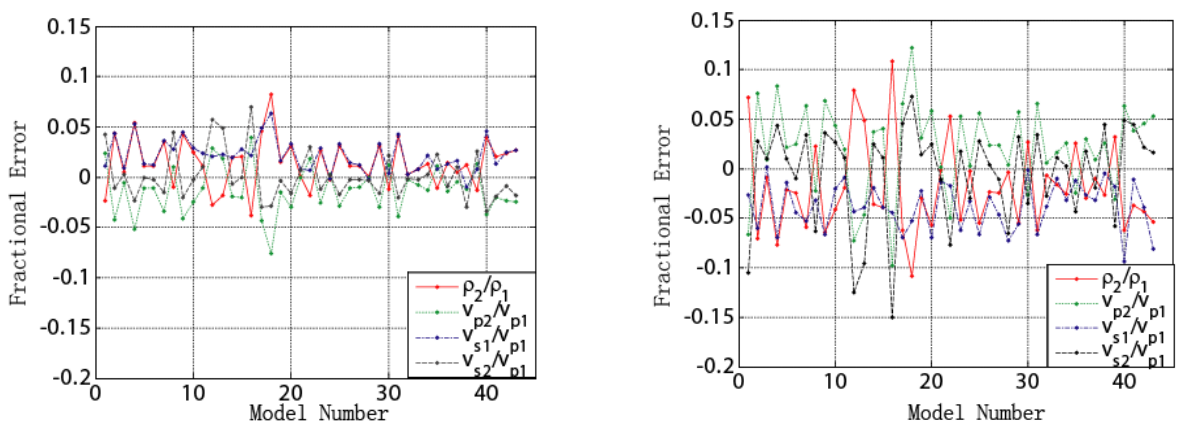


Figure 1 PP and PS AVO responses to model (Table 1)



(a) PP alone (b) PS alone (c) Joint PP and PS

Figure 2. Comparison of the object function value convergence between (a) PP alone ,(b) PS alone and (C) Joint PP and PS.



(a) (b)

Figure 3. The fractional error of model parameters obtained from joint PP and PS inversion. Model parameters are from Ramos(2001). Four independent fitting attributes (two independent attributes for PP and PS respectively) as input observations are obtain from (a) Power and (b) Polynomial fits to AVO curve.