
Hong Feng*
University of Calgary, Calgary, AB
hfeng@ucalgary.ca

Brian Russell
CGGVeritas, Calgary, AB, Canada

and

John Bancroft
University of Calgary, Calgary, AB, Canada

Summary
Numerous approaches have been published that derive fluid indicators (often called direct hydrocarbon indicators, or DHI) from AVO (Amplitude Variations with Offset) equations. The main idea behind these methods is to use the linearized Zoeppritz equations to extract petrophysical parameters such as P-impedance, S-impedance, bulk modulus, shear modulus, Lamé's parameters, and Poisson's ratio, and infer the fluid content from cross-plots of these parameters. Russell et al. (2003) used standard poroelasticity theory (Biot, 1941, and Gassmann, 1951) to generalize the several of these methods using a parameter dependent on the dry rock $V_p/V_s$ ratio. Also, In this study, we use the Han (1986) lab measurements to compare the generalized fluid method with other fluid methods.

Introduction
We found that the general linearized approximation for Zoeppritz's equation falls into two categories. One category linearly involves $V_p$, $V_s$, and $\rho$, and the other one is nonlinearly related to $V_p$, $V_s$, and $\rho$. For the first category, Aki and Richards equation emphasized the contribution of variations in the P- and S-wave velocities and density. A reformulation of the Aki-Richards approximation is the basis of many of the empirical amplitude variations with offset work performed today, and the three extracted parameters are called intercept, gradient, and curvature. Fatti et al. (1994) give us the weighted zero-offset P-wave and S-wave reflectivities and density reflectivity. Shuey (1985) transformed from $V_s$ and $\Delta V_s$ to Poisson's ratio $\sigma = (\gamma^2 - 2)/(2\gamma^2 - 2)$ and $\Delta \sigma$. Gray et al. (1999) showed how to estimate the parameters $K$, $\lambda$, $\mu$, and $\rho$ more directly by a new parameterization of the linearized AVO equation, as does Chen (1999). From the estimated P- and S-wave reflectivities of Fatti et al. (1994) we can invert for P- and S-Impedance. Goodway et al. (1997) developed the $\lambda \mu \rho$ method, in which these impedances were transformed to produce an estimate of $\lambda \rho$, a fluid
indicator, and $\mu \rho$, an indicator of the rock matrix. Russell et al. (2003) generalized the $\lambda \mu \rho$ method, introducing a constant scale factor dependent on the dry rock $V_p/V_s$ ratio as a better fluid discriminator. In this talk, we will first review the generalized fluid method and then test this method using a number of measured saturated and dry rocks.

**The Generalized Fluid Method**

As discussed by Russell et al. (2003), the seismic velocity of an isotropic material can be estimated using known rock moduli and density. P- and S-wave velocities in isotropic media are estimated as:

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + 4/3\mu}{\rho}}$$

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

where $V_p$ and $V_s$ are the P- and S-wave velocities, $K$ is bulk modulus, $\mu$ is shear modulus, $\lambda$ is the Lamé parameter, and $\rho$ is the mass density.

Based on Biot-Gassmann theory, we have the following relationship (Krief et al, 1990):

$$\lambda_{sat} = \lambda_{dry} + \beta^2 M$$

$$K_{sat} = K_{dry} + \beta^2 M$$

where $\lambda_{sat}$ is the first Lamé parameter for the saturated rock, $\lambda_{dry}$ is the first Lamé parameter for the dry frame, $K_{sat}$ is the bulk modulus of the saturated rock, $K_{dry}$ is the bulk modulus of the dry rock, $\beta$ is the Biot coefficient, and $M$ is the modulus or the pressure needed to force water into the formation without changing the volume.

Using the $\beta^2 M$ term, we can rewrite the equation for P-wave velocity (equation 1) in the saturated case as:

$$V_p = \sqrt{\frac{\lambda_{dry} + 2\mu + \beta^2 M}{\rho_{sat}}} = \sqrt{\frac{K_{dry} + 4\mu / 3 + \beta^2 M}{\rho_{sat}}}$$

or, as

$$V_p = \sqrt{(s + f) / \rho_{sat}}$$

where $f$ is a fluid term equal to $\beta^2 M$, and $s$ is a dry-skeleton term that can be written either as $K_{dry} + (4/3)\mu$ or $\lambda_{dry} + 2\mu$.

Using P- and S-wave impedances we can derive the generalized fluid term:

$$I_p^2 - cI_s^2 = \rho(s + f - c\mu)$$

Russell et al. (2003) suggested that proper selection of $C$ can cancel the dry skeleton term in the equation (7). If $C = \left(\frac{V_p}{V_s}\right)_{dry}^2$ equation (7) will be:

$$I_p^2 - cI_s^2 = \rho(s + f - c\mu) = \rho f$$

Murphy et al. (1993) and Hedlin (2000) have suggested that $K_{dry} / \mu$ is equal to 0.9 to derive $K_p$, which is equivalent to $\beta^2 M$ in equation (3). In their formulation, $C$ is equal to 2.233. Goodway et al.
(1997) suggested using $\lambda \rho$ as the hydrocarbon indicator, which implies a C value of 2. Hilterman (2001) assumes that $K_{\text{dry}} / \mu$ is equal to 1.0, which implies a C value of 2.333.

Inspired by the Grey et al. (1999) parameterization of the linearized AVO equation, Russell et al. (2006) re-expressed the Aki-Rechards equation as:

$$R_{pp}(\theta) = a(\Delta f / f) + b(\Delta \mu / \mu) + c(\Delta \rho / \rho) \quad (9)$$

where $a = (1/4 - \gamma^2_{\text{dry}} / 4\gamma^2_{\text{sat}}) \sec^2 \theta$, $b = \gamma^2_{\text{dry}} / 4\gamma^2_{\text{sat}} \sec^2 \theta - 2 / \gamma^2_{\text{sat}} \sin^2 \theta$, $c = 1/2 - 1/4 \sec^2 \theta$, $\gamma^2_{\text{sat}} = (V_p / V_s)^2_{\text{sat}}$, and $\gamma^2_{\text{dry}} = (V_p / V_s)^2_{\text{dry}}$, which is equivalent to to the c value in equation (8).

Equation (9) gives us new physical insight into the relationship between linearized AVO and poroelasticity, and is a generalization of the equations of Gray et al. (1999). If we use $\gamma^2_{\text{dry}} = 2$, we obtain the Gray et al. (1999) expression for $\lambda$, $\mu$, and $\rho$. If we use $\gamma^2_{\text{dry}} = 4/3$, we obtain the Gray et al. (1999) expression for $K$, $\mu$, and $\rho$. Russell et al. (2003) claim that $\gamma^2_{\text{dry}} = 2.333$, which implies that $K / \mu = 1$, is more appropriate for the reservoir rocks such as consolidated sandstones.

**Empirical Study**

To diagnose the sensitivity of the generalized fluid term with other fluid indicators, we use Han’s (1986) data set, which covers a wide range of porosities and clay contents at different pressure. These samples are used not only for calculating the fluid terms but also for investigating the effect of clay content, porosity, and pressure on the c value in sandstone.

From Figure 1, it is easy to observe that c increases with porosity and clay content. Also, the c value increases with the pressure. The value of 2.233 is appropriate for clean sands. For lithologies other than clean sandstone, c is about 2.333.

Table 1 shows the mean value, standard deviation, and the fluid indicator coefficient for each fluid term at the pressure of 5 MPa. Each fluid indicator coefficient diagnose the sensitivity to fluid discrimination and is defined as the difference between dry and wet sandstone divided by the mean value of the standard deviation for wet and dry sandstone. It can be observed that the fluid coefficient for $\lambda \rho$, $K$, $\lambda / \mu$, $\sigma$, $K-\mu$, $Ip^2_{\text{clis}}$, and $\Delta F$ are much higher than other indicators, which means these indicators can easily be used to separate wet sandstones from dry sandstones. Among these indicators, $Ip^2_{\text{clis}}$ is the most sensitive indicator for the fluid content. With these rigidity modulus related attributes, such as $V_s$, $I_s$, $\mu \rho$, and $\mu$, we have almost no chance of distinguishing dry sandstone from wet sandstone.

Figure 2 is the plot of the fluid indicator coefficient versus pressure. These lines suggest that, with decreasing pressure, we have a better chance to separate wet sandstone from dry sandstone. At the pressure of 5 MPa, the dry sandstone has significantly higher values of fluid indicator coefficient for indicators $\lambda \rho$, $K$, $\lambda / \mu$, $\sigma$, $K-\mu$, $Ip^2_{\text{clis}}$, and $\Delta F$ than those at 50 MPa. That means the chance to differentiate dry sandstone from wet sandstone is bigger at shallow depths, but is possible at greater depths.
Figure 1: (a) Plot of C value versus pressure; (b) Plot of C value versus porosity color-coded with clay content for 65 shaly sandstone samples at Pe = 5 MPa.

Table 1: Mean, Standard deviation, and fluid indicator coefficient for each fluid indicator at 50 MPa.

<table>
<thead>
<tr>
<th>Hydrocarbon indicators</th>
<th>Vp (km/s)</th>
<th>Vs (km/s)</th>
<th>Vp/Vs (·)</th>
<th>p (g/cc)</th>
<th>Ip (km/s.g/cc)</th>
<th>Is (km/s.g/cc)</th>
<th>Ip^2-C*Is^2 (GPa.g/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>Mean value</td>
<td>5.33</td>
<td>2.22</td>
<td>1.5</td>
<td>2.2</td>
<td>7.4</td>
<td>4.94</td>
</tr>
<tr>
<td></td>
<td>Std dev</td>
<td>0.75</td>
<td>0.49</td>
<td>0.08</td>
<td>0.2</td>
<td>2.07</td>
<td>1.41</td>
</tr>
<tr>
<td>Wet</td>
<td>Mean value</td>
<td>5.84</td>
<td>2.19</td>
<td>1.77</td>
<td>2.37</td>
<td>9.15</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>Std dev</td>
<td>0.66</td>
<td>0.48</td>
<td>0.12</td>
<td>0.13</td>
<td>1.8</td>
<td>1.34</td>
</tr>
<tr>
<td>Fluid indicator coefficient</td>
<td>0.70</td>
<td>0.06</td>
<td>4.50</td>
<td>0.85</td>
<td>0.85</td>
<td>0.20</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 2: Mean, Standard deviation, and fluid indicator coefficient for each fluid indicator at 50 MPa.

Figure 2: Fluid indicators coefficient versus pressure. Observe that the indicators, Ip^2 - C*Is^2, ΔK, ΔK-μ, Δλ are more effective than other indicators.
Conclusions

Various combinations of rock properties have been proposed as hydrocarbon indicators, and it can be concluded that a great deal of equivalence exists between fluid indicators $I_{p^2-cs^2}$, $K-\mu$, $\lambda\rho$, and $\lambda/\mu$. For sandstones, the difference $I_{p^2-cs^2}$ might be most sensitive in absolute terms. However, most of these indicators give similar results in magnitude and each can give insight into the meaning of the other. The best indicator needs to be calibrated and tested for local situations.

References

Hedlin, K., 2000, Pore space modulus and extraction using AVO: SEG Expanded Abstracts, 170-173.
Murphy, W., Reischer, A., and Hsu, K., 1993, Modulus decomposition of compressional and shear velocities in sand bodies: Geophysics, 58, 227–239.