AVO Modeling of Monochromatic Spherical Waves: Comparison to Band-Limited Waves

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Summary

Monochromatic and band-limited spherical waves have differing reflection coefficient curves. To facilitate comparison, a new expression for monochromatic reflectivity is given in terms of a weighting function. The weighting function approach, developed previously for a specific class of band-limited spherical waves (Rayleigh filtered waves), shows explicitly how different plane waves contribute to a spherical-wave reflection coefficient. Direct comparison shows that monochromatic waves have oscillatory, non-decaying weighting functions, and thus sample a wide range of plane waves. In contrast, typical Rayleigh wavelets produce localized weighting functions. These two behaviors lead to reflection coefficient curves which differ beyond the critical angle. A bridge between these two behaviors is constructed by considering unusually narrow Rayleigh wavelets. These show intermediate properties. This study shows 1) a simple and convenient method for calculating monochromatic spherical-wave reflection coefficients, and 2) a clearer understanding of how spherical-wave reflection coefficients are created from constituent plane-waves.

Introduction

Historically, the most common approach for describing reflectivity of spherical waves in seismic exploration has been through constructing the reflection coefficients for a monochromatic source. This approach is due to Lamb (1904) and Sommerfeld (1909) and is described in Aki and Richards (1980). Carrying out such calculations for multiple frequencies allows one to obtain the reflection coefficient for a band-limited wavelet via an inverse Fourier transform, as has been carried out by Haase (2004).

We have previously presented a direct approach to band-limited reflection coefficients (Ursenbach et al., 2005). This employs the Rayleigh wavelet (Hubral and Tygel, 1989), which allows the inverse Fourier transform to be carried out analytically. Only one numerical integral is then required to obtain the reflection coefficient for a given geometry. The final result is expressed as

$$R_{\mathrm{PP}}^{\text{spherical (Rayleigh)}}(\theta_i) = \int_\Gamma W_n(S_0, \theta, \theta_i) R_{\mathrm{PP}}(\theta)\, d(\cos\theta)\,, \tag{1}$$

where θ_i is the angle of incidence, θ is an integration parameter, Γ is an integration path in the complex plane, $S_0 \equiv \alpha_1 / (D\omega_0)$, α_1 is the P-wave velocity of the overburden, *D* is the length of the raypath from source to receiver, ω_0 is the dominant frequency of the Rayleigh wavelet, and W_n is a normalized weighting function, with *n* a parameter of the Rayleigh wavelet. For Rayleigh wavelets, W_n is an analytic function which can be readily programmed.

In addition to providing a speedy approach to calculating spherical-wave reflection coefficients, the Rayleigh wavelet approach of equation 1 also provides useful insight into the relationship between plane-wave and spherical-wave reflectivities. The W_n kernel is largest when θ and θ_i are similar, and decays rapidly when $|\theta - \theta_i|$ is large. Thus the spherical-wave reflection coefficient receives contributions primarily from plane-wave coefficients near the angle of incidence. Indeed, as $S_0 \to 0$, the spherical-wave coefficient approaches the plane-wave Zoeppritz result (Ursenbach et al., 2005).

To obtain a similar picture for single frequencies, we first derive an expression for the monochromatic spherical-wave reflection coefficient which is similar in form to equation 1. We then compare monochromatic reflection coefficients to band-limited reflection coefficients, and monochromatic weighting functions to band-limited weighting functions. We will demonstrate that the two cases differ significantly, but that the band-limited results approach the monochromatic results for increasingly narrow bands.

Theory

Analogous to equation 6.30 of Aki and Richards (1980), the monochromatic potential for a reflected spherical wave may be written as

$$\phi(\omega) = Ai\omega \exp(-i\omega t) \int_0^\infty \frac{p}{\xi} R_{\mathrm{PP}}(p) J_0(\omega p r) \exp[i\omega\xi(z+h)]\, dp \tag{2}$$

where $\phi(\omega)$ is the spectrum of the displacement potential, is frequency, *A* is an arbitrary scale factor, *t* is time, *p* and are horizontal and vertical slownesses ($\alpha_1 = 1/\sqrt{p^2 + \xi^2}$), R_{PP} is the plane-wave reflection coefficient, J_0 is a zeroth-order Bessel function, *r* is the source-receiver offset, and *z* and *h* are the vertical distances from the interface to the receiver and source.

To proceed from equation 2 to an expression for the spherical-wave reflection coefficient, we follow steps similar to those described in detail in Ursenbach et al. (2005) for the Rayleigh wavelet band-limited case: (1) Obtain the displacement spectrum from the gradient of the potential parallel to the ray vector at the receiver, $(\sin\theta_i, 0, \cos\theta_i)$, where θ_i is the angle of incidence. (2) Divide this result by the simple displacement spectrum obtained using R_{PP} = 1. (3) Perform a change of variables for the integration and define $\sin\theta_i = p\alpha_1$ and $\cos\theta_i = \xi\alpha_1$. (4) Set *h* = *z*, and note that

$z = (D/2)\cos\theta_i$ and $r = D\sin\theta_i$ (where $D = \sqrt{r^2 + 4z^2}$). (5) Note that the integrand now depends upon only three variables, θ, θ_i, and *S*, where *D*, and $_1$ appear only in the combination $S \equiv \alpha_1 / (D\omega)$, a quantity which provides a measure of the importance of curvature and spherical effects. The final result can then be written

$$R_{\mathrm{PP}}^{\text{spherical (monochromatic)}}(\theta_i) = \int_{\Gamma} W(S,\theta,\theta_i) R_{\mathrm{PP}}(\theta)\, d(\cos\theta)\,, \tag{3}$$

$$W(S,\theta,\theta_i) = \frac{\left[-J_1(\sin\theta\sin\theta_i / S)\sin\theta\sin\theta_i + iJ_0(\sin\theta\sin\theta_i / S)\cos\theta\cos\theta_i\right]}{S\left(1-iS\right)\exp\left[i(1-\cos\theta\cos\theta_i)/S\right]}\,. \tag{4}$$

We have thus derived an equation of the form of equation 1, and can compare reflection coefficients and weighting functions between monochromatic waves and band-limited waves.

Results

We consider a Class 1 AVO system defined in Table 1. We assume a frequency for the monochromatic wave of 100/*π* Hz (≈ 31.8 Hz), and an interface depth of 500 m, so *S* = .01cosθ_i. Equations 3 and 4 may then be solved to give the monochromatic reflection coefficient curve, while, given an appropriate wavelet, a band-limited result may be obtained using the method of Ursenbach et al. (2005). Recent developments have improved this method so that calculations may be readily carried out for Rayleigh wavelets with large values of *n*, where the Rayleigh wavelet is given by $w(f) = f^n \exp[-n(f/f_0)]$ and f_0 is the dominant frequency. If f_0 is set equal to the frequency of the monochromatic wave, then choosing an appropriate set of *n* values should provide a bridge between monochromatic behavior and that of a typical seismic wavelet. This is illustrated in Figure 1a, where the Rayleigh wavelet spectrum approaches a spike as $n \to \infty$. In Figure 1b we see a corresponding progression in the spherical-wave reflection coefficient curves. The monochromatic curve is highly oscillatory just past the critical angle while the band-limited solution approaches the plane-wave asymptote much more smoothly.

Table 1. Two-layer, elastic interface model employed in calculations

	Density (kg/m^3)	P-wave velocity (m/s)	S-wave velocity (m/s)
Layer 1	2400	2000	879.88
Layer 2	2000	2933.33	1882.29

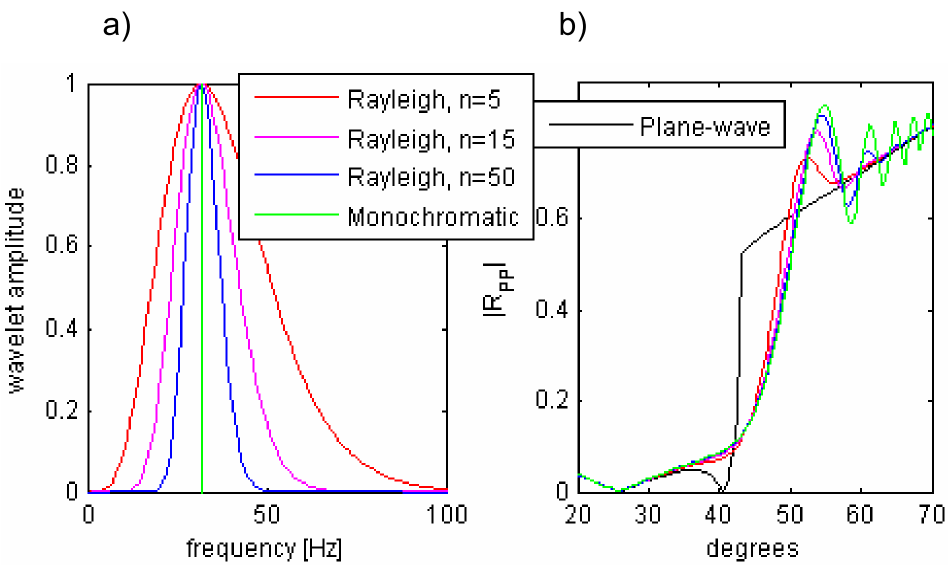

Figure 1. (a)
The spectra of three Rayleigh wavelets. As n increases, the spectrum becomes spike-like, so that its reflectivity behavior should approach that of a monochromatic wave-let.

Figure 1. (b)
The spherical-wave reflection coefficient curves for the wavelets in (a), and the corresponding plane-wave curve and mono-chromatic spherical-wave curves.

To understand these differences we consider the weighting functions that give rise to the above reflection coefficients. Figure 2 displays weighting functions for the three Rayleigh wavelets and the monochromatic wavelet for θ_i = 40°. The n = 15 and n = 50 wavelets do indeed form intermediates. They decay away from $\theta = \theta_i$, as does the n = 5 wavelet, but their decay is slower and more oscillatory, approaching the behavior of the monochromatic weighting function.

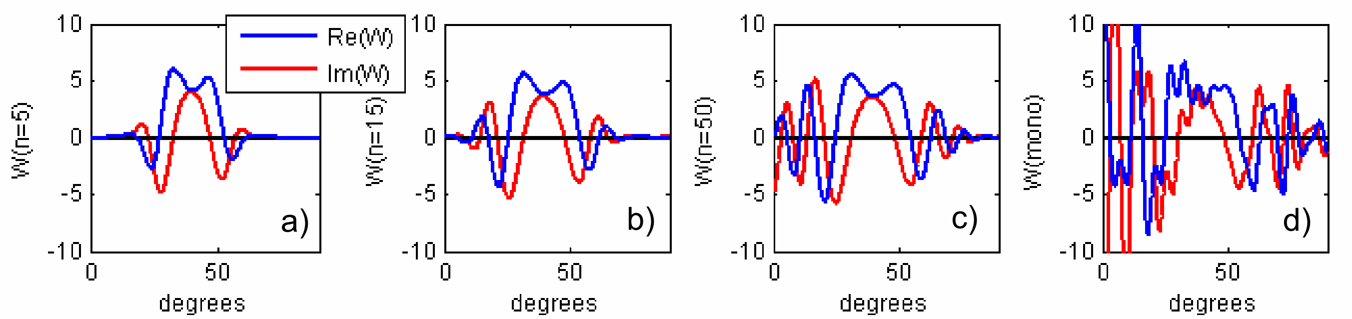

Figure 2. The weighting functions defined at θ_i = 40° for the wavelets in Figure 1a. Note that the tails of the Rayleigh wavelet weighting functions [a)-c)] become increasingly oscillatory as n grows, thus approaching the appearance of the monochromatic wavelet weighting function in d).

Thus framing calculations in terms of weighting functions, as in equations 1 and 3, provides insight into spherical-wave calculations for different types of wavelets.

Summary and Comments

Monochromatic spherical-wave reflection coefficient calculations have been re-expressed in terms of a weighting function. This weighting function depends explicitly on only three variables: angle of incidence, an integration variable, and a sphericity parameter. The latter subsumes frequency, overburden velocity and depth. The weighting function is analytic and may be readily programmed in terms of these three variables. A straightforward 1-D numerical integration then yields the normalized reflection coefficient.

Calculations with the method have shown that weighting functions for monochromatic wavelets are non-decaying and highly oscillatory. Comparing them to a series of weighting functions and reflection coefficient curves for increasingly narrow Rayleigh wavelets suggests that the less smooth the wavelet spectrum is, the more oscillatory the weighting function will be, and this will result in oscillations in the reflection coefficient curve as well.

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