NMO Inversion for Multilayer Subsurface with Horizontal Transverse Isotropic layers

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Introduction

The multilayered transverse isotropic model with horizontal anisotropy axis is used to study fractured reservoirs. Reflection moveout in fractured media is usually used to determine azimuth of main set of fractures and to estimate the crack density and to predict whether the ctacks are fluisd or dry (Contrera et al (1999)). In this paper, I use an inversion technique, originally developed by Blias (1983) for the layered medium with vertical symmetry axis (VTI media). This algorithm is utilized to determine an explicit formula for azimuthally-dependent NMO velocity in a HTI layered media with depth-varying orientation of the symmetry axis. The NMO inversion problem for the HTI medium was solved by Contrera et al (1999) using a technique, originally developed by Grechka and Tsvankin (1999). They derived a formula for azimuth-dependent NMO velocity using 2x2 matrices responsible for the 'interval' NMO ellipses in each layer. Here I suggest an alternative approach to NMO velocity derivation for the multilayered transverse isotropic model with depth-varying orientation of the symmetry axis. Explicit analytical formula for azimuth-dependent NMO velocity helps to develop insight on the anisotropy influence on reflection trveltime and to develop Dix-type inversion algorithm.

Method

Let's consider a layered medium with horizontal boundaries and transverse isotropic layers with horizontal anisotropy axis in each layer. Suppose the traveltimes of a reflected wave have been recorded for a fixed midpoint but for different azimuthal orientation. Let d be the source-receiver distance and d_x and d_y be the x and y progection of the sourse-receiver segment, α is an azimuth, fig. 1. Then the traveltime t is a function of d_x and d_y. This function contains only even powers of d_x and d_y. For conventional spreadlengths close to the reflector depth, for the aurbitrary 3-D subsurfaces, the traveltime function $t(d_x,d_y)$ can be written in a form (Blias, 1988):

$$t^2(d_x, d_y) = t_0^2 + w_{11}d_x^2 + 2w_{12}d_x d_y + w_{22}d_y^2 \qquad (1)$$

We can rewrite this equation using azimuth α ($d_x = d\cos\alpha$, $d_y = d\sin\alpha$)

$$t^2(d_x, d_y) = t_0^2 + \left(w_{11}\cos^2\alpha + 2w_{12}\sin\alpha\cos\alpha + w_{22}\sin^2\alpha\right)d^2 = t_0^2 + \frac{d^2}{V_{NMO}^2(\alpha)} \qquad (2)$$

This representtaion for anisotropic media was proposed by Grechka and Tsvankin (1998). Let h_k be the thickness of k-th layer, φ_k be the azimuth of the anisotropy axis in the same layer; v_{0k} is a vertical velocity in the layer. For each layer, we the HTI model is characterized by the stiffness tensor c_{ijkl} that corresponds. P and SV phase velocities are described with formulas using Thomsen's notations (Thomsen, 1986).

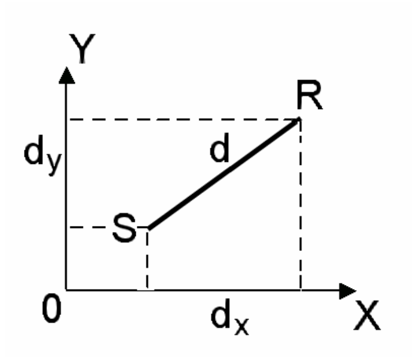

Figure 1. Source-receiver scheme

I am using the same approach that has been developed by Blias (1983) for VTI medium. The only difference is that in HTI layer, $\Theta=\pi/2$ corresponds to the vertical axis so instead of using angle θ, I will use an angle $\alpha = \pi/2-\theta$. For the phase P velocity $V_P(\alpha)$ and phase angle α, we can write equaitons:

$$\rho V_P^2(\alpha) = \alpha_0^2\left[1+\varepsilon\cos^2\alpha + \hat{D}(\alpha)\right],$$

$$\psi(\alpha) = \frac{\pi}{2} - \alpha - arctg\left(\frac{d\ln V}{d\alpha}\right),$$

where $D(\sin\theta) = \hat{D}(\cos\alpha)$, ε is Thomsen parameter (Thomsen, 1986). From these equations, we can find explicit connection between the group velocity v_P and group angle ψ::

$$V_P(\psi) = V_0 + \frac{1}{2}\frac{\partial^2 V_P}{\partial\psi^2}\left(\frac{\pi}{2}-\psi\right)^2 + \frac{1}{24}\frac{\partial^4 V_P}{\partial\psi^4}\left(\frac{\pi}{2}-\psi\right)^4 + ...$$

A reflaction traveltime T can be calculated as a sum of traveltimes in each layer:

$$T = \sum_{k=1}^{n} t_k(x_k, y_k) = 2\sum_{k=1}^{n}\frac{\sqrt{h_k^2 + x_k^2 + y_k^2}}{v_k(\theta_k(x_k, y_k))} \qquad (2)$$

Here x_k and y_k are X and Y the projections of the ray in the k-th layer,

$$\sum_{k=1}^{n} x_k = 2d_x,$$
$$\sum_{k=1}^{n} y_k = 2d_y, \qquad (3)$$

θ_k is a ray angle, determined by the equation:

$$\theta_k = \arccos\left(\frac{x_k \cos\varphi_k + y_k \sin\varphi_k}{\sqrt{h_k^2 + x_k^2 + y_k^2}}\right)$$

To find minimum of function T with respect to x_k, y_k, we use Lagrange function, which leads us to the system of equations:

$$\frac{\partial t_k(x_k, y_k)}{\partial x_k} = \lambda \qquad \frac{\partial t_k(x_k, y_k)}{\partial y_k} = \mu \qquad k=1, 2, \ldots, n \qquad (4)$$

where λ and μ are Lagrange multipliers (Blias, 1983). We have 2n+2 equations (3) – (4) to determine 2n+2 unknown x_k, y_k, λ and μ. First we find x_k and y_k as a solution of system (4) as functions of λ and μ independently for each k:

$$x_k = \lambda x_{1,k} + \mu x_{2,k} + \lambda^3 x_{3,k} + \ldots \qquad y_k = \lambda y_{1,k} + \mu y_{2,k} + \lambda^3 y_{3,k} + \ldots \qquad (5)$$

Then we substitute (5) into (2) and (1) and come to the parametric form of the time-distance relationship with λ and μ as parameters:

$$T = T_0 + T_{2,0}\lambda^2 + T_{1,1\lambda}\mu + T_{0,2}\mu^2 + \ldots$$
$$d_x = d_{1,x}\lambda + d_{2,x}\mu + d_{3,x}\lambda^3 + d_{4,x}\lambda^2\mu + \ldots$$
$$d_y = d_{1,y}\lambda + d_{2,y}\mu + d_{3,y}\lambda^3 + d_{4,y}\lambda^2\mu + \ldots$$

From these equations we can determine coefficients w_{ij} in formula (1):

$$w_{1,1} = \frac{1}{2} t_0 B_{11} \Big/ (B_{11}B_{22} - B_{1,2}^2), \qquad w_{1,2} = -\frac{1}{2} t_0 B_{12} \Big/ (B_{11}B_{22} - B_{1,2}^2),$$
$$w_{2,2} = \frac{1}{2} t_0 B_{22} \Big/ (B_{11}B_{22} - B_{1,2}^2) \qquad (6)$$

where

$$B_{11} = \sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \left(1 + 2\hat{\delta}_{P,k} \sin^2\varphi_k\right), \qquad B_{12} = 2\sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \hat{\delta}_{P,k} \cos\varphi_k \sin\varphi_k$$

$$B_{22} = \sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \left(1 + 2\hat{\delta}_{P,k} \cos^2\varphi_k\right) \qquad (7)$$

Here $\hat{\beta}_P$ is a coefficient, similarly to Thomsen parameter β and defined by formula:

$$2\hat{\delta}_P = \frac{(C_{13} - C_{11} + 2C_{44})(C_{13} + C_{11})}{(C_{11} - C_{44})C_{11}}$$

The same equation holds for SV waves, only $\hat{\delta}_{SV}$ is determined by formula:

$$2\hat{\delta}_{SV} = \frac{(C_{33} - C_{44})(C_{11} - C_{44}) - (C_{11} + C_{44})^2}{(C_{11} - C_{44})C_{44}}$$

If $\hat{\delta}_{P,k} = 0$ then formulas (6) – (7) are the same as for isotropic medium. Formulas (6) can be written in a matrix form:

$$\mathbf{B} = \mathbf{W}^{-1}$$

where **B** = $||B_{ij}||$, **W** = $||w_{ij}||$.

Inverse Problem

Now we describe Dix-type inversion to determine interval vertical velocities and anisotropic parametrs $\hat{\beta}_P$ and $\hat{\beta}_{SV}$. From traveltime measurement we know coefficients w_{11}, w_{12} and w_{22}. Then we consider equations (6) as a system with respect to A, B and C. This system has a unique solution:

$$B_{11,n} = \frac{1}{2}\frac{t_{0n} w_{1,1}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - \left(w_{1,2}^{(n)}\right)^2}, \quad B_{12,n} = -\frac{1}{2}\frac{t_{0n} w_{1,2}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - \left(w_{1,2}^{(n)}\right)^2}, \; B_{22,n} = \frac{1}{2}\frac{t_{0n} w_{2,2}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - \left(w_{1,2}^{(n)}\right)^2} \qquad (8)$$

where n is a reflector number. Taking into account (7), we can write system with unknown :

$$v_{0,n}^2 \Delta t_n \left(1 + 2\hat{\delta}_n \sin^2 \varphi_n\right) = 2\Delta B_{11,n},$$
$$v_{0,n}^2 \Delta t_n \hat{\delta}_n \cos\varphi_n \sin\varphi_n = \Delta B_{12,n},$$
$$v_{0,n}^2 \Delta t_n \left(1 + 2\hat{\delta}_n \cos^2 \varphi_n\right) = 2\Delta B_{22,n}$$

where $\Delta A_n = A_n - A_{n-1}$, $\Delta B_n = B_n - B_{n-1}$, $\Delta C_n = C_n - C_{n-1}$. This system can be easily solved for the vertical velocity $v_{0,n}$, azimuth φ_n and anisotropic parameter. After replacing $\hat{\beta}_P$ by $\hat{\beta}_{SV}$, formulas (6) – (7) hold for SV waves. It directly implies that we can find vertical SV velocity and anisotropic coefficient $\hat{\beta}_{SV}$ by solving system (8) where the matrix B is calculated for SV waves.

Conclusions

A new reperesentation of NMO velocity has been derived for the multilayered transverse isotropic model with horizontal anisotropy axis in each layer. Dix's type of inversion is applied to azimuth-dependent NMO velocity to determine vertical P and S velocities and anisotropic parameters $\hat{\beta}_P$ and $\hat{\beta}_{SV}$.

References

Blias, E., 1983, Reflected wave's travel-time curve in flat-bedded medium with transverse isotropic layers and their interpretation: Soviet Geology and Geophysics, 91-95.

Blias E., 1988, Reflected wave's traveltime in 3D layered media with curvilinear boundaries: Mathematic Problems of Seismic data interpretation, 96-128, Novosibirsk (in Russian).

Contreras, P., Grechka, V. and Tsvankin, I., Moveout inversion of P-wave data for horizontal transverse isotropy: Geophysics, 64, 1219-1229.

Grechka, V. and Tsvankin, I. 1998, 3-D description of normal moveout in anisotropic inhomogeneous media: *Geophysics* 63, 1079–1092.

Grechka, V., and Tsvankin, I., 1999, 3-D moveout velocity analysis and parameters estimation for orthorhombic media: Geophysics, **64**, 820–837.

Grechka V., Tsvankin I. and Cohen J. K., 1999, Generalized Dix equation and analytic treatment of normal-moveout velocity for anisotropic media: *Geophysical Prospecting*, 47, 117–148

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966